Applied Mathematics and Mechanics
(English Edition, Vol. 15, No. 10, Oct. 1994)

Published by SU,
Shanghai, China

THE MAXIMUM DISSIPATIVE EXTENSION OF SCHRÖDINGER OPERATOR*

Tian Li-xin (田立新)

(*Department of Mathematics and Physics, Jiangsu Institute of Technology, Zhengjiang, Jiangsu; LNM, Institute of Mechanics, Academia Sinica, Beijing*)

(Received Nov. 22, 1993; Communicated by Dai Shi-qiang)

Abstract

In the present paper we study the maximum dissipative extension of Schrödinger operator, introduce the generalized indefinite metric space and get the representation of maximum dissipative extension of Schrödinger operator in natural boundary space, make preparation for the further study longtime chaotic behavior of infinite dimension dynamics system in nonlinear Schrödinger equation.

Key words infinite dimension dynamics system, nonlinear Schrödinger equation, indefinite metric space, dissipative operator

I. Introduction

The study of longtime chaotic behavior of Schrödinger equation makes the study of soliton theory richer. Since nonlinear Schrödinger equation is very complex, these problems are difficult. One can see the works about these prolems in [1]−[4], especially, [1] studies attractors and fractal dimension of this equation. We study nonlinear Schrödinger equation using the methods developed recently in studying longtime behavior of infinite dimension dynamics system. In this paper we will resolve the first question of this problem, give the representation of maximum dissipative extension of Schrödinger operator in this equation. Then, using the representation, we will study the geometric form of attractor, temple chaotic, inertial manifold and inertial form is Schrödinger equation. Then, in the Section II, using the generalized semi-inner product space, we will give a new space: generalized indefinite metric space, which holds the meaning of Banach space in general. In the meantime, we give the natural boundary space of Schrödinger operator, which is a generalized indefinite metric space. Then, using the indefinite metric of natural boundary space, we give the one−one correspondence between the maximum dissipative extension of Schrödinger operator and the maximum negative subspace of natural boundary space. We give the representation.

According to [4], Schrödinger operator is $-h\Delta+V(x)$, defined in $C^\infty(M)$, M is C^∞

* Project supported by the National Natural Science Foundation of China

compact Riemann manifold. The Schrödinger operator has unique self adjective extension in Sobolev space $H^2(M)$. But, if the domain isn't Riemann manifold, the operator becomes complex and the study of the Schrödinger equation becomes difficult. In this paper, we study the domain of Banach space, and give the maximum dissipative extension of this operator. Suppose that

$$L_0 f = if'' - f$$
$$D(L_0) = \{f : f,\ f',\ f'' \in L^{p'}[0,\ 2\pi],\ f(0) = f(2\pi),$$
$$f'(0) = f'(2\pi),\ 1 < p' < \infty\}$$

L_0 is the type of Schrödinger operator which will be studied. Let $X = L^{p'}[0,\ 2\pi]$, make generalized semi-inner product $[\cdot\ ,\ \cdot]_p$ (see [5]), where

$$[f,\ g]_p = \int_0^{2\pi} f\bar{g}|g|^{p-2}dx, \qquad 1 < p < \infty,$$

p may be different from p'.

Obviously the norm in X is $\|f\| = [f,\ f]^{1/p}$, suppose $A: D(L_0) \to X$, $Af = if''$.

Let $G(L_0) = \{\{f,\ L_0 f\} : f \in D(L_0)\}$. In $X \times X$, construct $Q(\ \cdot\ ,\ \cdot\ )$

$$Q(f,\ g) = (fg)'(2\pi) - (fg)'(0)$$
$$= f'(2\pi)g(2\pi) - f'(0)g(0) + f(2\pi)g'(2\pi) - f(0)g'(0)$$

Let $\tilde{H} = \bar{H}_+ \oplus \bar{H}_-$, here

$$\bar{H}_+ = \overline{\mathrm{span}}\{f \in X,\ Q(f,\ f) \geqslant 0\},\quad \bar{H}_- = \overline{\mathrm{span}}\{f \in X,\ Q(f,\ f) \leqslant 0\}$$

Denote $\hat{H} = \tilde{H}/G(L_0)$. Construct $\hat{Q} = \hat{Q}_+ + \hat{Q}_-$ in $\hat{H}$, here

$$\hat{Q}_+(\hat{f}_+,\ \hat{g}_+) = Q(\hat{f}_+,\ \hat{g}_+)\mathrm{sign}Q(\hat{g}_+,\ \hat{g}_+), \qquad \hat{f}_+,\ \hat{g}_+ \in \bar{H}_+$$
$$\hat{Q}_-(\hat{f}_-,\ \hat{g}_-) = Q(\hat{f}_-,\ \hat{g}_-)(-\mathrm{sign}Q(\hat{g}_-,\ \hat{g}_-)), \qquad \hat{f}_-,\ \hat{g}_- \in \bar{H}_-$$

Let $\check{H} = \{f : f,\ f' \in X\}$, $\bar{H} = \hat{H} \oplus \check{H}$, for any $\tilde{f} \in \{\hat{f},\ \check{f}\}$, define $\tilde{Q}$

$$\tilde{Q}(\tilde{f},\ \tilde{g}) = \hat{Q}(\hat{f},\ \hat{g}) + [\check{f},\ \check{g}]_p.$$

Theorem 1 A is symmetric operator in Banach space X. (see[5] for the definition)

Theorem 2 $(\bar{H}_+/G(L_0),\ \hat{Q}_+)$, $(\bar{H}_-/G(L_0),\ -\hat{Q}_-)$, are generalized semi-inner product space.

Theorem 3 $(\hat{H},\ \hat{Q})$ is generalized indefinite metric space.

Theorem 4 If the maximum dissipative extension of L_0 is L, then L responds one to one with the maximum negative subspace $\tilde{N}$ of $\bar{H}$ and.

$$Lu = iu'' - u + \varphi(\hat{u})$$
$$D(L) = \{u : u,\ u',\ u'' \in X,\ \hat{u} \in \hat{N}\},$$

$\hat{N}$ is projection of $\tilde{N}$ from $\bar{H}$ to $\hat{H}$.

II. The Proof of the Theorem

In this paper, operator L_0 is in Banach space, naturally, the study of the operator L_0 is more difficult than that of operators in Hilbert space. For this reason, we construct generalized semi-inner product in Banach space and use generalized p self adjective, dissipative operator [5]. The study of indefinite metric space can be seen in [6] and [7] . Now we construct generalized indefinite metric space. Further and deeper study of this space will be given elsewhere in another paper.

Definition 1 R is complex (or real) linear space. Define $\langle y, z\rangle$ for arbitrary $x, y, z \in R$, $\lambda \in C$(complex), if the following is satisfied

(1) $\langle x+y, z\rangle=\langle x, z\rangle+\langle y, z\rangle$, $\langle \lambda x, y\rangle=\lambda\langle x, y\rangle$.

(2) For arbitrary $y\in R$, if $\langle x, y\rangle=0$, then $x=0$.

Thus we call $(R, \langle\cdot,\cdot\rangle)$ generalized Krein space.

Definition 2 Space $(R, \langle\cdot,\cdot\rangle)$ is called generalized indefinite metric space, if it includes two subspace H_+, H_- and

(1) $R=H_+ \oplus H_-$, here $\oplus$ is direct sum and orthogonal according to $\langle\cdot,\cdot\rangle$ that is $\langle x, y\rangle=0$, $x\in H_+$, $y\in H_-$.

(2) $(H_+, \langle\cdot,\cdot\rangle)$, $(H_-, -\langle\cdot,\cdot\rangle)$ are generalized semi-inner product space.

Here $H_+=\{x\in R|\langle x, x\rangle\geqq 0\}$, $H_-=\{x\in R|\langle x, x\rangle 0\leqq\}$, H_+, H_- are called positive subspace and negative subspace, respectively.

Lemmal 1 R is generalized indefinite metric space.

If $x, y\in R$, $x=x_++x_-$, $y=y_++y_-$, $x_+, y_+\in H_+$, $x_-, y_-\in H_-$. Let

$$[x, y]=\langle x_+, y_+\rangle+\langle x_-, y_-\rangle,$$

then$(R, [\cdot,\cdot])$ is a generalized semi-inner product space.

Proof The main point is to prove the following inequality

$$|[x, y]|\leqslant[x, y]^{1/p}[y, y]^{(p-1)/p}, \qquad p>1.$$

Since $ab\leqslant a^p/p+b^q/q, a>0, b>0$, here $1/p+1/q=1$, it is sufficient to prove.

$$(1+km^{p-1})^p\leqslant(1+k^p)(1+m^p)^{p-1}$$

The above inequality and be proved using series Young inequality. Omit the proof.

Define project operator $P_\pm : \Pi=(R, \langle\cdot, \cdot\rangle)\to H_\pm$, so that $x=x_++x_-\in\Pi\mapsto x_\pm\in H_\pm$.

Lemma 2 The necessary and sufficient for nonnegative subspace L to be maximum nonnegative subspace is that $P_+L=H_+$. Every nonnegative subspace is included in a maximum nonnegative subspace.

Omitted proof.

The proof of theorem 1

Assume $f, g\in D(L_0)$, then

$$[Af, g]_p=\int_0^{2\pi} if''g|g|^{p-2}dx=\int_0^{2\pi} if''g\left((p-2)\int_0^{|g|} a^{p-3}da\right)dx$$

$$=\int_0^{2\pi} if''g(p-2)\left(\int_0^{\infty} X_{[0,\ |g|]}(x)a^{p-3}da\right)dx$$

$$=(p-2)\int_0^{\infty} a^{p-3}\left(\int_0^{2\pi} igX_{[|g|>a]}(x)df'\right)da$$

$$=(p-2)\int_0^{\infty} a^{p-3}\left(-\int_0^{2\pi} ig'X_{[|g|>a]}(x)df\right)da$$

$$=(p-2)\int_0^\infty a^{p-3}\int_0^{2\pi} f\overline{(-ig^p)}X_{[|g|>a]}(x)dxda$$

$$=-(p-2)\int_0^\infty a^{p-3}\int_0^{2\pi} f\overline{(ig^p)}X_{[|g^p|>a]}(x)dxda$$

$$=-\int_0^{2\pi} f\overline{(ig^p)}\int_0^{|g^p|} d(a^{p-2})dx$$

$$=-\int_0^{2\pi} f\overline{(ig^p)}|g^p|^{p-2}dx$$

$$=-[f,\ Ag],$$

Hence A is symmetric operator.

The proof of theorem 2

As $\bar{f}\in G(L_0)$, thus $\bar{Q}_\pm(\bar{f},\ f)=0$ and $G(L_0)$ is Kernel space.

Now proving that $\bar{Q}_+(\cdot,\cdot)$ is generalized semi-inner product space, only to prove

$$|\bar{Q}_+(\bar{f},\ \bar{g})|\leqslant|\bar{Q}_+(\bar{f},\ \bar{f})|^{1/p}|\bar{Q}_-(\bar{g},\ \bar{g})|^{(p-1)/p},\qquad \bar{f},\ \bar{g}\in\bar{H}_\pm.$$

In fact,

$$|\bar{Q}_+(\bar{f},\ \bar{g})|=|Q(\bar{f},\ \bar{g})|=|\mathrm{Re}[A^*f,\ g]|$$
$$=0.5|\mathrm{Re}[A^*f,\ g]+\mathrm{Re}[f,\ A^{**}g]|$$

As $A^*\supset -A$, $-A^{**}\subset A^*$, then

$$|\bar{Q}_+(\bar{f},\ \bar{g})|=0.5|\mathrm{Re}[A^*f,\ g]+\mathrm{Re}[f,\ -A^*g]|$$

Set a new generalized semi-inner product in $H\times H$ following that

$$[\bar{u},\ \bar{v}]_{12}=\mathrm{Re}[u^1,\ v^1]+\mathrm{Re}[u^2,\ v^2],$$
$$\bar{u}=\{u^1,\ u^2\},\ \bar{v}=\{v^1,\ v^2\}\in H\times H.$$

Similary lemma 1, $[\cdot,\cdot]_{12}$ is generalized semi-inner product in $H\times H$. Let

$$W\bar{u}=W\{u^1,\ u^2\}=\{u^2,\ u^1\},\ \bar{u}\in H\times H.$$

Then W is generaliged p self-adjoint operator in $(H\times H,[\cdot,\cdot]_{12})$. Let

$$W_1\bar{u}=W\bar{u},\ \bar{u}\in H_+;\ W_1\bar{u}=-W\bar{u},\ \bar{u}\in\bar{H}_+\backslash H_+,$$

W_1 is positive operator, and easily proves the following inequality of W_1

$$|[W_1\bar{u},\ \bar{v}]_{12}|\leqslant|[W_1\bar{u},\ \bar{u}]_{12}|^{1/p}|[W_1\bar{v},\ \bar{v}]_{12}|^{(p-1)/p}.$$

And easily proves that

$$|Q_+(f,\ \bar{g})|\leqslant 0.5|[Wu',\ u']_{12}|^{1/p}|[Wv',\ v']_{12}|^{(p-1)/p}$$
$$=0.5\{|\mathrm{Re}[A^*f,\ f]_p+\mathrm{Re}[f,\ A^{**}f]_p|\}^{1/p}\{|\mathrm{Re}[A^*g,\ g]_p$$
$$+\mathrm{Re}[g,\ A^{**}g]_p|\}^{(p-1)/p}$$
$$=|\bar{Q}_+(\bar{f},\ \bar{f})|^{1/p}|\bar{Q}_-(\bar{g},\ \bar{g})|^{(p-1)/p}$$

where

$$u'=\{f,\ A^*f\},\ v'=\{g,\ A^*g\}.$$

Thus completes the proof.

Lemm 3 $\hat{Q}(\hat{f},\ \hat{g})=\bar{Q}(\bar{f},\ \bar{g})$, $\hat{f},\ \hat{g}$ is coset of $\bar{f},\bar{g}$. $\hat{f},\ \hat{g}\in\hat{H},\ \bar{f},\ \bar{g}\in\bar{H}$.

Proof Obviously $\hat{Q}(\hat{f},\hat{f})=0$, when $\hat{f}\in G(L_0)$. As $\hat{f}\in\hat{H}, \hat{f}_0\in G(L_0)$, first we prove $\hat{Q}(\hat{f}+\bar{f}_0,\ \hat{f})=\hat{Q}(\hat{f},\ \hat{f})$. In fact

$$\hat{Q}(\hat{f}+\bar{f}_0,\ \hat{f})=\bar{Q}_+(\bar{f}_++\bar{f}_0,\ \bar{f}_+)+\bar{Q}_-(\bar{f}_-,\ \bar{f}_-) \tag{2.1}$$

$$\begin{aligned}\bar{Q}_+(\bar{f}_++\bar{f}_0,\ \bar{f}_+)&=Q(\bar{f}_++\bar{f}_0,\ \bar{f}_+)\operatorname{sign}Q(\bar{f}_+,\ \bar{f}_+)\\&=[A^*(f_++f_0),\ f_+]\operatorname{sign}Q(\bar{f}_+,\ \bar{f}_+)\\&=[A^*f_+,\ f_+]\operatorname{sign}Q(\bar{f}_+,\ \bar{f}_+)+[Af_0,\ f_+]\operatorname{sign}Q(\bar{f}_+,\ \bar{f}_+)\\&=\bar{Q}_+(\bar{f}_+,\ \bar{f}_+)+[Af_0,\ \bar{f}_+]\operatorname{sign}Q_-(\bar{f}_+,\ \bar{f}_+),\end{aligned}$$

where

$$\bar{f}_+=\{f_+,\ L_1f_+\}\in\bar{H}_+,\ L_1=A^*-I,\ \bar{f}_0=\{f_0,L_0,f_0\}\in G(L_0).$$

Now prove $[A\ .f_0,\ f_+]=0$.

From Theorem 2, similary we have

$$\begin{aligned}|[Af_0,\ f_+]|&=|Q(\bar{f}_0,\ \bar{f}_+)|=0.5|[Wf_0',\ f_+']_{12}|\\&\leqslant 0.5|[Wf_0',\ f_0']_{12}|^{1/p}|[Wf_+',\ f_+']_{12}|^{(p-1)/p}\\&=0.5|[Af_0,\ f_0]_p|^{1/p}|[Wf_+',\ f_+']_{12}|^{(p-1)/p}=0,\end{aligned}$$

where

$$f'=\{f,\ A^*f\},\ \bar{f}_0=\{f_0;L_0,f_0\},\ \bar{f}_+=\{f_+,\ L_1f_+\}$$

Hence $[Af_0\ ,f_+]=0$

$$\begin{aligned}Q(\hat{f}+\bar{f}_0,\ \hat{f})&=\bar{Q}(\bar{f}+\bar{f}_0,\ \bar{f})=\bar{Q}_+(\bar{f}_+,\ \bar{f}_+)+\bar{Q}_-(\bar{f}_-,\ \bar{f}_-)\\&=\bar{Q}(\bar{f},\ \bar{f}).\end{aligned}$$

The following we prove

$$\begin{aligned}&\hat{Q}(\hat{f},\ \hat{f}+\bar{f}_0)=\bar{Q}(\bar{f},\ \bar{f}+\bar{f}_0)=\bar{Q}(\bar{f},\ \bar{f}),\\&\bar{f}\in\bar{H}_+\oplus\bar{H}_-,\ \bar{f}_0\in G(L_0),\end{aligned}$$

where $\bar{f}=\bar{f}_++\bar{f}_-$.

Remark $\bar{f}_++\bar{f}_0\in\bar{H}_+,\ \bar{f}_-+\bar{f}_0\in\bar{H}_-$.

$$\begin{aligned}\bar{Q}(\bar{f},\ \bar{f}+\bar{f}_0)&=\bar{Q}_+(\bar{f}_+,\ \bar{f}_+\bar{f}_0)+\bar{Q}_-(\bar{f}_-,\ \bar{f}_-)\\&=\bar{Q}_+(f_+,\ \bar{f}_+)+\ \ \bar{Q}_-(\bar{f}_-,\ \bar{f}_-+\bar{f}_0).\end{aligned}$$

As

$$\begin{aligned}\bar{Q}=(\bar{f}_+,\ \bar{f}_++\bar{f}_0)&=\bar{Q}_+(\bar{f}_+,\ \bar{f}_+)+\bar{Q}_-(0,\ \bar{f}_0)=\bar{Q}_+(\bar{f}_+,\ \bar{f}_+)\\&=\bar{Q}_+(\bar{f}_+,\ \bar{f}_++\bar{f}_0)=\bar{Q}(\bar{f}_+,\ \bar{f}_++\bar{f}_0),\end{aligned}$$

then

$$\bar{Q}_+(\bar{f}_+,\ \bar{f}_++\bar{f}_0)=\bar{Q}_+(\bar{f}_+,\ \bar{f}_+)$$

Similarly we have

$$\bar{Q}_-(f_-, \bar{f}_- + \bar{f}_0) = \bar{Q}_-(\bar{f}_-, \bar{f}_-),$$

hence

$$\bar{Q}(\bar{f}, \bar{f}+\bar{f}_0) = \bar{Q}_+(\bar{f}_+, \bar{f}_+) + \bar{Q}_-(\bar{f}_-, \bar{f}_-) = \bar{Q}(\bar{f}, \bar{f})$$

Now we get that

$$\hat{Q}(\hat{u}, \hat{u}) = \bar{Q}(\bar{u}, \bar{u}), \quad \hat{u} \in \bar{H}_+/G(L_0) \oplus \bar{H}_-/G(L_0), \quad \bar{u} \in \bar{H}_+ \oplus \bar{H}_-,$$

$\hat{u}$ is coset of $\bar{u}$.

The proof of theorem 3

As $\bar{Q}$ is indefinite metric and $\bar{Q}$ is semi-linear, we have that $(\hat{H}, \hat{Q})$ is generalized indefinite metric space, That is all.

Let $\bar{N}$ is negative subspace of $(\bar{H}, \bar{Q})$ and $\hat{N}$ is projection from $\bar{N}$ to $\hat{H}$, where $\bar{H} = \hat{H} \oplus \check{H}$. Prove easily that $\hat{N}$ is negative subspace of $(\hat{H}, \hat{Q})$ and

$$\|\check{u}\|^p \leqslant -\hat{Q}(\hat{u}, \hat{u}) = -\bar{Q}(\bar{u}, \bar{u}) \leqslant c\|\hat{u}\|^p, \quad \text{for any } \{\hat{u}, \check{u}\} \in \bar{N}.$$

Lemma 4 If $\bar{N}$ is maximum negative subspace, $\hat{N}$ is a maximum negative subspace of $\hat{Q}$

Lemma 5 If $\bar{N}$ is maximum negative subspace thus

$$\|\check{f}\|^p \leqslant -\hat{Q}(\hat{f}, \hat{f}), \quad \bar{f} = \{\hat{f}, \check{f}\} \in \bar{N}.$$

Define $\varphi: \hat{f} \mapsto \check{f}$, then φ is contraction mapping from $\hat{N}$ to $\check{H}$. Another, if φ is contraction mapping, then the graph of φ is maximum negative subspace in $\bar{H}$.

The proof of theorem 4

Assume that L is the maximum dissipative extension, then $\mathrm{Re}[Lf, f] \leqslant 0$, $f \in D(L)$.

If $f \in D(L_0)$, $g \in D(L)$, then

$$\mathrm{Re}[L_0 f, g] = \mathrm{Re}[Af, g] - [f, g]$$

$$|\mathrm{Re}[Af, g]| \leqslant |\mathrm{Re}[L_0 f, g]| + |[f, g]| \leqslant (\|L_0 f\| + \|f\|)\|g\|^{p-1}$$

$$|[Af, g]| \leqslant 2(\|L_0 f\| + \|f\|)\|g\|^{p-1}$$

Hence, $g \in D(A^*)$. $D(L_1) \supset D(L)$.

For arbitrary $g \in D(L_0)$, $f \in D(L)$, we have

$$|[g, Lf - L_1 f]| \leqslant \|g\| \|Lf - L_1 f\|^{p-1}.$$

From the Riesz representation theorem of generalized semi-inner product space [6], there exists unique $\check{f} \in \check{H}$, so that for any $g \in D(L_0)$,

$$[g, Lf - L_1 f] = [g, \check{f}], \quad Lf - L_1 f = \check{f}.$$

Therefore $Lu = L_1 u + \check{f}$. The following proves that

$$\hat{Q}(\hat{f}, \hat{g}) = \bar{Q}(\bar{f}, \bar{g}) = \bar{Q}_+(\bar{f}_+, \bar{g}_+) + \bar{Q}_-(\bar{f}_-, \bar{g}_-), \qquad \hat{f}, \hat{g} \in \hat{H}$$

$\hat{f}$, $\hat{g}$ are coset of $\bar{f}$, $\bar{g}$, and

$$\bar{f} = \bar{f}_+ + \bar{f}_-, \quad \bar{g} = \bar{g}_+ + \bar{g}_-, \qquad \bar{f}_+, \bar{g}_+ \in \bar{H}_+, \ \bar{f}_-, \bar{g}_- \in \bar{H}_-.$$

The following inequality will holds:

$$Q(\tilde{f}, \tilde{f}) - m\|f-\check{f}\|^p + \|\check{f}\|^p \leqslant 0 \tag{2.2}$$

here m is an arbitrary constant, $\tilde{f}=\{f, L_1 f\} \in G(L_0)$, $\check{f}$ is coset of $\tilde{f}$. First, proves

$$Q(\tilde{f}, \tilde{f}) - m\|f-\check{f}\|^p + \|\check{f}\|^p \leqslant 0, \qquad \tilde{f} \in G(L_1) \tag{2.3}$$

Since $Q(\tilde{f}, \tilde{f}) = [L_1 f, f] - [f, f]$, it follows that

$$\mathrm{Re}[L_1 f, f] = Q(\tilde{f}, \tilde{f}) - [f, f].$$

As

$$\mathrm{Re}[Lf, f] \leqslant 0, \ \mathrm{Re}[L_1 f + \check{f}, f] \leqslant 0$$
$$Q(\tilde{f}, \tilde{f}) - \mathrm{Re}[f, f] + \mathrm{Re}[\check{f}, f] \leqslant 0.$$

Hence, to prove (2.3), we only need to prove

$$-m\|f-\check{f}\|^p + \|\check{f}\|^p \leqslant -[f, f] + \mathrm{Re}[\check{f}, f]. \tag{2.4}$$

In fact

$$\begin{aligned}
&\mathrm{Re}[-f+\check{f}, f] \\
&\leqslant |[-f+\check{f}, f]| \leqslant \|f-\check{f}\|\|f\|^{p-1} \\
&\leqslant \|f-\check{f}\|^p/p + \|f\|^q/q, \qquad 1/p+1/q=1
\end{aligned}$$

The left hand of the above equality $[f, f] + \mathrm{Re}[\check{f}, f]$. So

$$[f, f] - \mathrm{Re}[\check{f}, f] \geqslant -\|f-\check{f}\|^p/p - \|f\|^p/q$$

Using the above inequality in (2.4), we have to prove

$$-m\|f-\check{f}\|^p + \|\check{f}\|^p - \|f-\check{f}\|^p/p - \|f\|^p/q \leqslant 0$$
$$\text{or} \quad (-m-1/p)\|f-\check{f}\|^p + \|\check{f}\|^p \leqslant \|f\|^p/q \tag{2.5}$$

Note that

$$\begin{aligned}
&\mathrm{Re}[-f+\check{f}, \check{f}] \\
&\quad \leqslant |[-f+\check{f}, \check{f}]| \leqslant \|\check{f}-f\|\|\check{f}\|^{p-1} \leqslant \|f-\check{f}\|^p/p + \|\check{f}\|^q/q \\
&q\mathrm{Re}[-f+\check{f}, \check{f}] \leqslant (q/p)\|f-\check{f}\|^p + \|\check{f}\|^p \\
&\quad -q\mathrm{Re}[f, \check{f}] + (q-1)\|\check{f}\|^p \leqslant q\|f-\check{f}\|^p/p \\
&\quad -(q/(q-1))\mathrm{Re}[f, \check{f}] + \|\check{f}\|^p \leqslant (q/(p(q-1)))\|f-\check{f}\|^p \\
&\quad -(q/(q-1))\mathrm{Re}[f, \check{f}] \leqslant (q/(p(q-1)))\|f-\check{f}\|^p - \|\check{f}\|^p
\end{aligned}$$

Hence,

$$\begin{aligned}
&\|\check{f}\|^p - \|f-\check{f}\|^p \leqslant p\mathrm{Re}[f, \check{f}] \leqslant p\|f\|\|\check{f}\|^{p-1} \\
&\leqslant p[(1/p)\|f\|^p + (1/ql)\|\check{f}\|^{(p-1)q}] \\
&= l\|f\|^p + (p/ql)\|\check{f}\|^p
\end{aligned}$$

In the above formula, let $l>0$ and $1-(p/ql)>0$.

Take $0<l<(\sqrt{1+4p}+1)/q$, then $l^2-l/q-p/q^2<0$, $1-(p/ql)>0$.

The above form also used the following inequality

$$ab \leqslant la^p/p + b^q/ql, \quad 1/p + 1/q = 1, \quad a>0, \quad b>0, \quad l>0.$$

By simplifying, we get

$$(1-p/ql)\|\check{f}\|^p - \|f-\check{f}\|^p \leqslant l\|f\|^p$$

or

$$\|\check{f}\|^p - \|f-\check{f}\|^p/[1-p(ql)^{-1}] \leqslant l\|f\|^p/[1-p(ql)^{-1}]$$

Using it in (2.5), we only have to prove

$$(-m-1/p+1/[1-p(ql)^{-1}])\|f-\check{f}\|^p \leqslant (-l/(1-p(ql)^{-1})+1/q)\|f\|^p \qquad (2.6)$$

To prove (2.6), take $m=-1/p+1/[1-p(ql)^{-1}]<0$. At this time, the left hand of (2.6) is 0, the coefficient the right hand $=-l/[1-p(ql)^{-1}]+1/q>0$. Therefore, (2.6) holds naturally. Hence (2.3) holds. So

$$Q(\tilde{f},\tilde{f}) - m\|f-\check{f}\|^p + \|\check{f}\|^p \leqslant 0, \quad \tilde{f}=\{f, L_1 f\}, m<0.$$

It follows that, $Q(\tilde{f},\tilde{f})\leqslant 0$, $\tilde{f}\in \tilde{H}_-$, $\bar{Q}(\tilde{f},\tilde{f})=\bar{Q}_-(\tilde{f},\tilde{f})$. So $\hat{Q}(\hat{f},\hat{f})=\bar{Q}(\tilde{f},\tilde{f})=\bar{Q}_-(\tilde{f},\tilde{f})$. Hence (2.2) holds.

Since $Lv=L_1v-L_0v$, $v\in D(L_0)$, $\check{f}$ depends on the coset of $\hat{f}$ in $\hat{H}$, $\check{f}=\varphi(\hat{f})$.

Note that $-m<0$, so $-m\|f-\check{f}\|^p\geqslant 0$. Hence, the middle term of equality (2.2) may be omitted and

$$\hat{Q}(\hat{f},\hat{f})+\|\varphi(\hat{f})\|^p\leqslant 0, \qquad f\in D(L_0)$$

Therefore $\{\hat{f}, \varphi(\hat{f})\}$, $f\in D(L_0)$ forms a negative subspace corresponding to $\tilde{Q}$ in $\tilde{H}$.

On the other hand if $Lu=L_1u+\varphi(\hat{u})$ and L is the extension of L_0 and $\{\{\hat{u},\varphi(\hat{u})\}\mid u\in D(L)\}$ is the maximum negative subspace of $\tilde{H}$, then it can be shown that (2.2) holds, so L is p dissipative operator.

Therefore, there exists an one to one correspondence negative subspace of $\tilde{H}$. The representation is gotten. This completes the proof.

References

[1] Yan Yin, Attractors and dimensions for discretizations of a weak damped Schrödinger equation and a Sine-Gorden equation, *Nonlinear Analysis, TMA*, **20**, 12 (1993), 1417—1452.

[2] Soffer, A., Multichannel nonlinear scattering for nonintegrable equations, *J. Differential Equation*, **98** (1992), 376—390.

[3] Hayashi, Nakao, The initial value problem for the derivative nonlinear schrödinger equation in the energy space, *Nonlinear Analysis, TMA*, **20**, 7 (1993), 823—833.

[4] Helffer, B., Semi-classical analysis for the Schrödinger operator and applications, *Lecture Notes in Math.*, **1336** (1988).

[5] Wei Guo-qiang and Shen You-qiang, The generalized p normal operators and p hyponomal operators on Banach spaces, *Chinese Ann. Math. Ser B*, **8**, 1 (1987), 70—79.

[6] Langer, H., Spectral functions of definitzable operators in Krein spaces, *Lecture Notes in Math.*, **948** (1984), 1—46.

[7] Bonger, L., Indefinite inner product spaces, *Ergeb. Math. Grenzgeb.* **78** (1974)