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ON THE ORIGIN OF INERTIA

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(Received 1952 August 20)*

Summary

As Einstein has pointed out, general relativity does not account satisfactorily for the inertial properties of matter, so that an adequate theory of inertia is still lacking. This paper describes a theory of gravitation which ascribes inertia to an inductive effect of distant matter. In the rest-frame of any body the gravitational field of the universe as a whole cancels the gravitational field of local matter, so that in this frame the body is " free ". Thus in this theory inertial effects arise from the gravitational field of a moving universe. For simplicity, gravitational effects are calculated in flat space-time by means of Maxwell-type field equations, although a complete theory of inertia requires more complicated equations.

This theory differs from general relativity principally in the following respects :

(i) It enables the amount of matter in the universe to be estimated from a knowledge of the gravitational constant.

(ii) The principle of equivalence is a consequence of the theory, not an initial axiom.

(iii) It implies that gravitation must be attractive.

The present theory is intended only as a model. A more complete, but necessarily more complicated theory will be described in another paper.

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1. *Introduction.*—In this paper we construct a tentative theory to account for the inertial properties of matter. These properties imply that at each point of space there exists a set of reference frames in which Newton's laws of motion hold good—the so-called "inertial frames". If other frames are used, Newton's laws will no longer hold unless one introduces "fictitious" (inertial) forces which depend on the motion of these frames relative to an inertial frame.

The question then arises: what determines the inertial frames? Newton asserted that they were determined by absolute space. However, absolute space is not observable in any other way, and it has been suggested that it is more satisfactory to attempt to correlate the inertial frames with observable features of the universe. In particular, Berkeley (1) and Mach (2) maintained that inertial frames are those which are unaccelerated relative to the "fixed stars", that is, relative to a suitably defined mean of all the matter in the universe. This statement is usually known as Mach's principle. As this principle will be used as a guide in constructing our theory, we shall first discuss its general implications.

The view that the problem of motion can be completely discussed in terms of observables implies that a kinematical description of all the relative motions in the universe completely specifies the system, so that kinematically equivalent motions must be dynamically equivalent. For instance, the statement that the Earth is rotating and the universe is at rest should lead to the same dynamical

* Received in original form 1952 January 3.

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consequences as the statement that the universe is rotating and the Earth is at rest, whereas this is not true in a scheme based on absolute space. Using Mach's principle we can predict that the angular velocity of the Earth, as deduced from a local dynamical experiment (such as the motion of a Foucault pendulum), will be the same as that deduced kinematically from the apparent motion of the fixed stars. This prediction cannot be made in Newton's theory, because there is then no causal connection between the motion of the stars and the existence of inertial forces at the Earth; the two observations give the same result only because, as it happens, the stars are not rotating relative to absolute space.

If the rest of the universe determines the inertial frames, it follows that inertia is not an intrinsic property of matter, but arises as a result of the interaction of matter with the rest of the matter in the universe. This immediately raises the problem of how Newton's laws of motion can be so accurate despite their complete lack of reference to the physical properties of the universe, such as the amount of matter it contains. It was largely this problem which originally prevented the general acceptance of Mach's ideas, and one of the requirements of a theory of inertia that is consistent with Mach's principle is that it should account for the *apparent* irrelevance of the properties of the universe.

The observed fact that a gravitational force is *locally* indistinguishable from an inertial force, in that each induces the same acceleration in all bodies, suggested to Einstein that it is the gravitational influence of the whole universe which gives rise to inertia. General relativity was devised to incorporate this idea, but, as emphasized by Einstein (**3**, **4**, *cf.* **5**, p. 97), it failed to do so. Einstein showed that his field equations imply that a test-particle in an otherwise empty universe has inertial properties. In view of this it seems to be worth while searching for theories of gravitation which imply that matter has inertia only in the presence of other matter. In this paper we describe what appears to be the simplest possible theory of gravitation that has this property, though this theory is incomplete in other respects.

2. *General formalism.*—Our problem is to construct a formalism in which the motion of a body is influenced by the presence of other bodies, but in which the concepts of "inertia" and "inertial frames" do not have to be introduced *a priori*. We shall represent the influence of the bodies on each other by a set of quantities defined at all points of space and time. As we are ignoring electromagnetic effects in this paper, we say that these quantities describe the gravitational field. The field is determined by the bodies (the sources) by means of a set of differential equations, together with suitably chosen boundary conditions. These equations show how the field can be determined from the motion (and other properties) of the sources.

In addition we must know how the field affects the motion of the sources. For this purpose we introduce the following postulate: *in the rest-frame of any body the total gravitational field at the body arising from all the other matter in the universe is zero.* In Newtonian language we could say that the universe moves relative to any body in such a way that the body never experiences a force—the difference from the ordinary Newtonian theory being that the forces acting on the body are here derived entirely from the matter in the universe.

We must now set up the equations relating the gravitational field to its sources. In the rest-frame of any particle we assume the field to be derivable from a potential in Minkowski space, that is, we do not describe it in terms of a curved

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space. Kinematical considerations (Section 6) show that the potential should be a tensor of the second rank, but the use of such a potential leads to rather involved mathematics which tends to obscure the physical significance of the theory. It seems advisable to begin by working with the simplest mathematical scheme which contains the physically important aspects of the problem. The simplest type of potential we could use is a scalar, but as we shall see (Section 4 (ii)), this would not give rise to inertia. The next simplest possibility is a vector potential, and with a theory based on such a potential we can reproduce the main properties of inertia. In this paper we shall confine our attention to such a potential. This simplification is useful for gaining insight into the problem, but naturally it has its limitations, some of which are mentioned in Section 6. The more elaborate equations that are needed for a tensor potential will be described in a subsequent paper (hereafter called II).

In a theory based on a vector potential, the field is an antisymmetrical tensor —the curl of the potential. The only linear second-order differential tensor equations for a field of this type that imply the conservation of source are (**6**) Maxwell's equations, which accordingly we shall adopt. We emphasize that although our equations have the same formal structure as Maxwell's, they describe purely gravitational effects, electromagnetic phenomena being outside the scope of this paper.

In order to apply the theory to even as simple a problem as the motion of a particle in the gravitational field of the Earth, we must know the distribution of matter in the universe. In practice we shall have to approximate to this distribution in some way. The type of approximation that will be most useful depends on the relative importance of near and distant matter. Since the amount of matter at a given distance increases roughly with the square of the distance, it follows that if the influence of matter falls off more slowly than the inverse square of the distance, then very distant matter is of predominant importance. It is convenient to anticipate that this is indeed the case (Section 4(iii)). This means that a smoothed-out model of the universe should be a good first approximation, local irregularities having only a small effect which can easily be estimated. It also means that local phenomena are strongly coupled to the universe as a whole, not just to local conditions. This in turn means that local experiments, if interpreted by means of this theory, can give us information about the structure of the universe as a whole (*cf.* **5, 7**). The correctness of this information can, in principle, be tested by independent and more direct considerations.

We shall take as our smoothed-out model a homogeneous and isotropic distribution of matter of density ρ expanding (relative to any point as origin) according to the Hubble law $\mathbf{v} = \mathbf{r}/\tau$, where $\mathbf{v}$ is the velocity of matter at distance $\mathbf{r}$, and τ is a constant. This neglects certain relativistic difficulties such as the significance of velocities exceeding that of light, but for the tentative theory developed in this paper we shall not concern ourselves with these problems; a consistent treatment will be given in II. This model is one of those in which there is a natural state of rest at each point, namely, that in which the observed distribution of the red-shifts of distant matter is isotropic. Thus we can speak of a body being at rest relative to the universe.

3. *Inertia-induction.*—In order to show how inertia arises in this formalism, we consider the behaviour of a test-particle in the presence of a single body

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superposed on the smoothed-out universe. The problem is to determine what motion of the system universe-plus-body relative to the test-particle makes the total gravitational field at the particle zero.

It is convenient to begin by calculating the potential at a test-particle that is at rest in a universe containing no irregularities. Since our field equations have the same form as Maxwell's, we can use electrodynamic formulae to calculate the potential, and to bring out the analogy with electrodynamics we use a similar notation and terminology, but we emphasize that in this paper we shall be concerned with purely gravitational phenomena.

Retardation effects are taken to arise in the same way as in electrodynamics, so that the contribution of any region of the universe to the potential at a point P at time t is computed by ascribing to that region just the properties that are observed at P at time t.

We thus have for the scalar potential (**8**)

$$\Phi = -\int_V \frac{\rho}{r} dV. \tag{1}$$

We use the minus sign in (1) because inertial mass then turns out to be positive, but in fact either sign can be used (Section 4 (vii)). The vector potential $\mathbf{A}$ vanishes by symmetry.

We shall assume that matter receding with velocity greater than that of light makes no contribution to the potential, so that the integral in (1) is taken over the spherical volume of radius $c\tau$. An assumption of this sort is necessary since we have naïvely extrapolated the Hubble law without considering relativistic effects, and should give the correct order of magnitude. A relativistic treatment is given in II.

Since the density is supposed uniform, (1) gives

$$\Phi = -2\pi\rho c^2\tau^2. \tag{2}$$

Owing to our assumptions, the numerical factor 2π is only approximate.

We now calculate the potentials for the simple case when the particle moves relative to the smoothed-out universe with the small rectilinear velocity $-\mathbf{v}(t)$. In the rest-frame of the particle the universe moves rectilinearly with velocity $\mathbf{v}(t)$. Now at time t there will be observable at the particle, in addition to the Hubble effect, a Doppler shift corresponding to $\mathbf{v}(t)$ from all parts of the universe. Hence, in computing the potential in the rest-frame of the particle at time t, we must ascribe to every region of the universe the velocity that is observed at time t, that is, $\mathbf{v}(t) + \mathbf{r}/\tau$.

Neglecting terms of order v^2/c^2, we have

$$\Phi = -2\pi\rho c^2\tau^2$$

as before. The vector potential no longer vanishes, but has the value

$$\mathbf{A} = -\int_V \frac{\mathbf{v}\rho}{cr} dV. \tag{3}$$

Since $\mathbf{v}$ is independent of $\boldsymbol{r}$, we can take it outside the integral. We then obtain

$$\mathbf{A} = \frac{\Phi}{c}\mathbf{v}(t).$$

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Since the change of ρ with time is very small, the gravelectric part of the field is approximately

$$\mathbf{E} = -\operatorname{grad}\Phi - \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t}$$

$$= -\frac{\Phi}{c^2}\frac{\partial \mathbf{v}}{\partial t},$$

while the gravomagnetic field is

$$\mathbf{H} = \operatorname{curl}\mathbf{A} = 0.$$

So far we have been concerned with a universe that has no irregularities. We now suppose that a body of gravitational mass M is superposed on this universe and is at rest relative to it. The field of this body in the rest-frame of the test-particle is then

$$-\frac{M}{r^2}\hat{\mathbf{r}} - \frac{\phi}{c^2}\frac{\partial \mathbf{v}}{\partial t}, \tag{4}$$

where r is the distance of the body from the test-particle, $\phi(= -M/r)$ is the potential of the body at the test-particle, and

$$\hat{\mathbf{r}} \,.\, \frac{d\mathbf{v}}{dt} = \frac{dv}{dt}.$$

The total field at the particle is zero if

$$-\frac{M}{r^2} - \frac{\phi}{c^2}\frac{dv}{dt} = \frac{\Phi}{c^2}\frac{dv}{dt}$$

or

$$\frac{M}{r^2} = -\left(\frac{\Phi+\phi}{c^2}\right)\frac{dv}{dt}. \tag{5}$$

This equation asserts that the system universe-plus-body accelerates relative to the test-particle at a rate determined by the mass and distance of the body. In accordance with the discussion of Section 1, we may re-interpret this result by saying that the particle accelerates towards the body, which is at rest relative to the universe, and thereby make contact with the Newtonian view-point. Indeed we obtain for rectilinear motion *a combination of Newton's laws of motion and of gravitation, with the inertial frame determined by Mach's principle.*

Furthermore, the gravitational constant satisfies the equation

$$\frac{\Phi+\phi}{c^2} = -\frac{1}{G}$$

or, since $\phi \ll \Phi$ (*cf.* Section 4 (iii)),

$$G\Phi = -c^2. \tag{6}$$

4. *Consequences of the theory.*—(i) Equation (6) implies that the total energy (inertial plus gravitational) of a particle at rest in the universe is zero. It can in principle be tested observationally, for, in conjunction with (2), it implies that

$$2\pi G\rho\tau^2 = 1$$

or

$$G\rho\tau^2 \sim 1, \tag{7}$$

since the numerical factor is only approximate.

Equation (6) implies that the gravitational constant at any point is determined by the total gravitational potential at that point, and so by the distribution of

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matter in the universe. This illustrates the fact mentioned above that if local phenomena are strongly coupled to the universe as a whole, then local observations can give us information about the universe as a whole. With our assumption about the structure of the universe, a laboratory determination of G, combined with an astronomical determination of τ, enables us to deduce the mean density of matter in space (apart from the uncertainty in the numerical constant).

Taking $G = 6 \times 10^{-8}$ c.g.s. units and $\tau = 6 \times 10^{16}$ sec we have from (7)

$$\rho \sim 5 \times 10^{-27} \text{ g cm}^{-3}. \tag{8}$$

A mean density of this amount is much larger than the usual observational estimates ($\sim 10^{-30}$ g cm^{-3}), but these values refer only to matter condensed into nebulae, and the mean density of internebular material may well exceed the overall density of material condensed into nebulae (**5**, p. 45). Indeed, if the nebulae are supposed to have condensed from an internebular medium, there is no reason why the medium should have been exhausted in the process. It is probable that ρ has an upper limit of $\sim 10^{-25}$ g cm^{-3}, as this appears to be the mean density within some nebulae. Hence (8) is not inconsistent with observation.

(ii) We see from the argument leading to (5) that "inertia-induction" arises from the term $\partial\mathbf{A}/\partial t$, that is, from the "radiation-field" of the universe. Had we used a scalar potential such a term would not have arisen, there would have been no inertia-induction, and we would not have been able to obtain Newton's law of motion (5).

(iii) The contribution of matter to local inertia falls off only inversely as the distance, since $\partial\mathbf{A}/\partial t$ is proportional to the scalar potential (*cf.* the radiation field of an accelerating charge (**8**, p. 22)). This means that the main contribution comes from distant matter—(1) shows that 99 per cent of local inertia arises from matter further away than 10^8 light-years. The fractional contributions of the Earth, Sun and Milky Way are 10^{-9}, 10^{-8}, 10^{-7} respectively (taking the mass of the Milky Way as 10^{44} g); this justifies the neglect of ϕ in (6). According to our theory, then, local phenomena are strongly coupled to the universe as a whole, but owing to the small effect of local irregularities this coupling is practically constant over the distances and times available to observation. Because of this constancy, local phenomena appear to be isolated from the rest of the universe.

(iv) As has already been pointed out, a theory of inertia that is consistent with Mach's principle must account for the fact that Newton's methods were so successful despite their complete lack of reference to the properties of the universe. The work of Section 3 shows that our theory satisfies this requirement. The universe affects local phenomena at just the two points where Newton's work contains arbitrary elements, namely (*a*) in the choice of inertial frames, and (*b*) in the value of the gravitational constant.

(v) Relation (7) is a consequence of many general relativistic models (**5**, *cf.* also **9**) as well as of our theory. However, our theory is disproved if (6) disagrees with observation, whereas general relativity is not, since it leads to many models in which (6) is false. Hence in general relativity the observed value of the gravitational constant gives no information about the amount of matter in the universe. For instance, if the Milky Way was thought to be the sole occupant of the universe, general relativity would still be consistent with the observed value of G, whereas in our theory there would be a discrepancy of a factor $\sim 10^7$. Indeed, if we accepted Mach's principle for the reasons given

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in Section 1, we would, on the basis of our theory, *predict* that the universe contained vastly more matter than had yet been observed.

(vi) The principle of equivalence asserts that the phenomena occurring in a gravitational field are the same as those occurring in the absence of the field if observations are made from a suitably chosen accelerating frame. For instance, consider an observer confined to a closed laboratory. According to the principle of equivalence the observer cannot distinguish between the following two events:

(*a*) a gravitating mass is suddenly placed near the laboratory;

(*b*) the laboratory is suddenly accelerated, e.g. by being pulled by a rope.

In our theory this is explained by the fact that in case (*b*) *the motion of the universe relative to the observer produces the same gravitational field at the laboratory as that of the mass in case* (*a*) (*cf.* the argument leading to (5)).

General relativity has difficulty in explaining the principle of equivalence because it predicts that one gravitating mass in an otherwise empty universe produces effects similar to those calculated with a full universe in Section 3. Since there is no universe to act as the source of the field in case (*b*), it is difficult to see why the principle of equivalence should be true.

(vii) It follows from our equations that gravitation is attractive, whereas in general relativity the sign of the field is not determined. This result arises because our equations imply that the field at a point P due to a mass is *decreased* in absolute magnitude if the mass has a component of acceleration towards P, and *increased* if it has a component away from it, whichever the sign of the field (*cf.* (4)). Thus the field of the universe can only cancel the field of a local mass if the universe moves so that the mass, which is at rest relative to it, accelerates towards the particle. This is why the sign in equation (1) is arbitrary: had we chosen a plus sign, gravitation would have been repulsive and inertial mass negative (corresponding to changing the sign of both sides of (5)), so that the particle would still accelerate towards the body.

5. *Uniform rotation.*—In Section 3 we assumed that the universe-plus-body moved in a straight line relative to the test-particle. We shall now derive another possible motion of the system in which the universe and body rotate with constant angular velocity about an axis through the centre of the body perpendicular to the line joining it to the particle.

In the rest-frame of the particle we set up a Cartesian system of axes with origin at the body and z-axis along the axis of rotation. If the universe were non-rotating, its gravitational potential near the origin would be

$$A_x = A_y = A_z = 0; \quad \Phi = -1, \tag{9}$$

choosing units so that $G = c = 1$. If it rotates, its potential in the x, y plane near the origin is

$$\begin{aligned} A_x &= \omega y, \\ A_y &= -\omega x, \\ A_z &= 0; \\ \Phi &= -[1+\omega^2(x^2+y^2)]^{1/2} \\ &= -(1+\omega^2 r^2)^{1/2}. \end{aligned}$$

This follows from the transformation properties of four-vectors (**10**). It is only true for distances from the origin for which the potential of a non-rotating

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universe differs negligibly from (9), since Rosen's result was derived for a four-velocity which has the values (9) everywhere in the non-rotating frame. This restriction is of no importance for phenomena on a galactic scale since the potential of the universe at the centre of the galaxy differs from that at the edge by less than one part in 10^{10}.

We thus get

$$\mathbf{E} = -\operatorname{grad}\Phi - \frac{\partial \mathbf{A}}{\partial t}$$
$$= \frac{\omega^2 \mathbf{r}}{(1+\omega^2 r^2)^{1/2}}$$
$$\sim \omega^2 \mathbf{r}, \quad \text{for} \quad \omega r \ll 1.$$

The field of the body, neglecting its rotation, is

$$-\frac{M}{r^2}\hat{\mathbf{r}}.$$

The total field is zero if

$$\frac{M}{r^2} = \omega^2 r. \tag{10}$$

In the rest-frame of the universe, this is the usual Newtonian equation for circular motion. In the rest-frame of the particle, however, Newton's law of motion only holds if we introduce a fictitious centrifugal force-field. Equation (10) shows that in our theory this field is not fictitious, but arises from the gravitational effect of a rotating universe, in agreement with Mach's principle.

In contrast to the case of Section 3, the gravomagnetic field is not zero (*cf.* the magnetic field of a rotating charge-distribution). In fact, we have

$$\mathbf{H} = \operatorname{curl}\mathbf{A} = 2\boldsymbol{\omega}.$$

Since the test-particle is at rest, this field has no effect; but for another test-particle constrained to move relative to the first with uniform velocity $\mathbf{v}$, there would be in its rest-frame a gravelectric field

$$\mathbf{v}\wedge\mathbf{H} = 2\mathbf{v}\wedge\boldsymbol{\omega} \tag{11}$$

acting on it. This can be seen directly by taking this particle to be at rest and determining $\mathbf{E}$ in the new frame. (*Cf.* the relativistic deduction of the Lorentz force $e(\mathbf{E}+\mathbf{v}\wedge\mathbf{H})$ from the expression $e\mathbf{E}$.)

The field (11) just corresponds to the Coriolis field of Newtonian theory, and like the centrifugal field it is here ascribed to the rotating universe. Thus, in our theory, we can regard the Earth as stationary and a Foucault pendulum as pulled around by the gravomagnetic field of the rotating universe.

Another feature of this interpretation of the Coriolis field is that the bending of light in a gravitational field is observable in the laboratory. In fact, the experiments of Sagnac (**11**) and Michelson and Gale (**12**) showed that in a rotating frame of reference light does not travel in straight lines. If we interpret this phenomenon according to our theory, we may suppose the frame to be at rest and the universe to be rotating around it. The resulting gravitational field then bends the path of the light. According to this point of view the Sagnac effect is of the same nature as the refraction of light passing through the gravitational field of the Sun. A quantitative treatment of the motion of light in a gravitational field requires, however, the more elaborate equations of II.

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The results of this section enable us to reconsider the problem of absolute rotation from the standpoint of our tentative theory. In fact, rotation does not give rise to any problems not already raised by translation, but the conflict between absolute and relative motion has usually been discussed in terms of rotation. The main argument for the view that there is an absolute standard of rotation has been the fact that local experiments can detect rotation—experiments in which the rest of the universe has not appeared to play an essential part. The present theory answers this argument by showing how the rest of the universe *can* play an essential part in local phenomena. The results of local experiments can be interpreted as giving us information about the universe as a whole—an interpretation which is verifiable by direct observation.

6. *Limitations of the theory.*—We have so far derived Newton's law of motion for two special cases: rectilinear motion and uniform rotation. A satisfactory theory of inertia must, of course, derive Newton's law for all possible motions. Now it follows from the relativistic form of Newton's law (**13**) that for general motions inertial forces are derived from a tensor potential and not from a vector potential. This means that the gravitational field of the moving universe, which in our theory gives rise to inertial forces, must also be derived from a tensor potential, so that our present theory is incomplete.

Furthermore, in a theory based on a vector potential it is difficult to give a consistent relativistic discussion of the structure of the universe as a whole. It is also difficult to describe the motion of light in a gravitational field. In paper II we shall describe a theory, using a tensor potential, in which these difficulties do not arise.

I am grateful to my colleagues for commenting on the manuscript, to the referees for their valuable and detailed criticisms of the first draft of this paper, and to Mr H. Bondi and Mr T. Gold for their constant advice and encouragement.

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1952 *August* 19.

References

(1) G. Berkeley, *The Principles of Human Understanding*, 1710; *De Motu*, 1726.
(2) E. Mach, *History and Root of the Principle of the Conservation of Energy*, 1872; *The Science of Mechanics*, 6th ed., 1904.
(3) A. Einstein, *S.B. preuss. Akad. Wiss.*, p. 142, 1917.
(4) A. Einstein, *The Meaning of Relativity*, p. 94 ff., Methuen, 1946.
(5) H. Bondi, *Cosmology*, Cambridge, 1952.
(6) H. Weyl, *Am. J. Maths.*, **66**, 591, 1944.
(7) H. Bondi and T. Gold, *M.N.*, **108**, 252, 1948.
(8) W. Heitler, *The Quantum Theory of Radiation*, Sect. 3, Oxford, 1944.
(9) G. J. Whitrow and D. G. Randall, *M.N.*, **111**, 455, 1951.
(10) N. Rosen, *Phys. Rev.*, **71**, 54, 1947.
(11) G. Sagnac, *J. de Phys.*, **4**, 177, 1914.
(12) A. A. Michelson and H. G. Gale, *Ap. J.*, **61**, 140, 1925.
(13) A. S. Eddington, *The Mathematical Theory of Relativity*, Sect. 15, 28, Cambridge, 1930.

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