The three Millennium problem solutions (RH, NSE, YME) and a Hilbert scale based geometrodynamics

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Jan 2019

This homepage addresses the problem& solution areas

A. The Riemann Hypothesis (RH)
B. The 3D-Navier-Stokes equations (NSE)   navier-stokes-equations.com
C. The Yang-Mills equations (YME)   quantum-gravitation.de
D. Loop Quantum Theory (LQT)   C. Rovelli (RoC), (RoC1)

building on a proposed common mathematical framework. The proposed solutions are building on common mathematical solution concepts and tools, e.g. Hilbert scale, Hilbert (resp. Riesz) transform(s), Hilbert-Polya conjecture. The proposed mathematical framework is also suggested to build a unified quantum field and gravity field theory, while applying the proposed mathematical framework to the Loop Quantum Theory (LQT).

The story line of this homepage is as follows:

an appropriate Hilbert scale framework is proposed to enable several RH criteria, while its correspondence to classical function spaces of the current Zeta function theory is ensured. One of the RH critera is about the Hilbert-Polya criterion, which is about a convolution operator representation of the Zeta function requiring a properly defined domain. This Hilbert space based operator representation with its underlying Hilbert scale frame (defining also the regularity of the Zeta funtion) can be applied to answer the Berry-Keating conjecture.

The Berry-Keating conjecture puts the zeros of the Zeta function (on the critical line, if the RH is true) in relationship to the (energy level) eigenvalues associated with the classical Hermitian operator H(x,p)=x*p ~ x*(d/dx), where x denotes the position coordinate and p the conjugate momentum. The Friedrichs extension of the variational representation of the Zeta function (on the critical) with L(2)-test space indicated a H(-1/2) quantum state space with related H(1/2) energy space. The today's standard quantum state resp. energy spaces are H(0)=L(2) resp. H(1), i.e. the latter Hilbert spaces are compactly embedded subspaces of the proposed new ones. Applying the physical quantum (fluid) Hilbert (state) space H(-1/2) to the 3-D non-linear, non-stationary NSE enables a well posed variational representation of the NSE with appropriate valid energy inequality, closing the Serrin gap problem. The correspondingly variational representation of the Maxwell equations enables a quantum field model (modified YME), enabling a differentiation of "elementary particles" with and w/o mass (modelled by the orthogonal decomposition of the Hilbert spaces H(-1/2) = H(0) + H(0,ortho) resp. H(1/2) = H(1) + H(1, ortho). It enables the concept of orthogal projection, which can be interpreted as "mass generation process during observation". Purely energy interaction of the "EP" are "acting" in the orthogonal space (which might be interpreted as zero point energy pool, "wave package resp. eigen-differential space). The macroscopic and microscopic state of quanta relate to corresponding frequencies of its vibrations. The corresponding action variables of the system ((HeW) II.1.c) define the related kinematical (physical) and thermodynamical concept of "*time*" ((RoC), (SmL)).(RoC1), section 13, "the source of time": "*Our interaction with the world is partial, which is why we see it in blurred way. To this blurring is added quantum indeterminacy. The ignorance that follows from this determines the existence of a particular variable - thermal time - and of an entropy that quantifies our uncertainty. Perhaps we belong to a particular subset of the world that interacts with the rest of it in such a way that this entropy is lower in one direction of our thermal time.*"

If in this context a successfully applied least action principle (being interpreted as a maxime of Kant's reflective judgment) results into appropriate consistent mathematical-physical models, those models can be declared as law of natures. The above is related to the three "forces of nature" as modelled by the SMEP. The nature of those elementary particles and the way they move, is described by quantum mechanics, but quantum mechanics cannot deal with the curvature of space-time. Space-time are manifestations of a physical field, the gravitational field. At the same time, physical fields have quantum character: granular, probabilistic, manifesting through interactions. The to be defined common mathematical solution framework needs to provide a quantum state of a gravitational field, i.e. a quantum state of space. The crucial difference between the photons characterized by the Maxwell equations (the quanta of the electromagnetic field) and the to be defined quanta of gravity is, that photons exists in space, whereas the quanta of gravity constitute space themselves ((RoC2) p. 148). The proposed mathematical framework provides a common baseline to integrate quantum mechanics & thermodynamics with gravity & thermodynamics. From a physical model problem perspective this is about a common mathematical framework for black body radiation ((BrK4) remark 2.6, Note O55, O71, O72) and black hole radiation ((RoC3) p. 56, 60 ff)). The thermodynamics is the common physical theory denominator with the Planck concept of zero point energy of the harmonic quantum oscillator (BrK), (BrK1), and the Boltzmann entropy concept. An integrated model needs to combine the underlying Bose-Einstein and the Dirac-Fermi statistic. In this context already Schrödinger suggested half-odd quantum numbers rather than integers. "*From the point of analogy one would very much prefer to do so. For, the "zero point energy" of a Planck oscillator is not only borne out by direct observation in the case of crystal lattices, it is also so intimitely linked up with the Heisenberg uncertainty relation that one hates to dispense with it. On the other hand, if we adopt it straightaway, we get into serious trouble, especially on contemplating changes of the volume (e.g. adiabatic compression of a given volume of black-body radiation*" ((ScE) p. 50).

This OVERVIEW page is frequently updated; a former related (more mathematical formula based) version (status August 2018) is provided in

Braun K., Three Millennium problems, RH, NSE, YME, OVERVIEW

A Kummer function based RH solution concept(s)

In order to prove the **Riemann Hypothesis** (RH) the Polya criterion can not be applied in combination with the Müntz formula ((TiE) 2.11). This is due to the divergence of the Müntz formula in the critical stripe due to the asymptotics behavior of the baseline function, which is the **Gaussian function**. The conceptual challenge (not only in this specific case) is about the not vanishing constant Fourier term of the Gaussian function and its related impact with respect to the Poisson summation formula. The latter formula applied to the Gaussian function leads to the Riemann duality equation ((EdH) 1.7). A proposed alternative "baseline" function than the Gaussian function, which is its related Hilbert transform, the **Dawson function**, addresses this issue in an alternative way as Riemann did. As the Hilbert transform is a convolution integral in a correspondingly defined distributional Hilbert space frame it enables the **Hilbert-Polya conjecture** (e.g. (CaD))**.** The corresponding distributional ("periodical") Hilbert space framework, where the Gaussian / Dawson functions are replaced by the fractional part / log(2sin)-functions enables the **Bagchi** reformulation of the **Nyman-Beurling RH criterion**.

The corresponding formulas, when replacing the Gaussian function by its Hilbert transform, are well known: the **Hilbert transform** of the **Gaussian** is given by the **Dawson integral** (GaW). Its properties are e.g. provided in ((AbM) chapter 7, (BrK4) lemma D1). The Dawson function is related to a special **Kummer function** in a similar form than the (error function) erf(x)-function resp. the li(x)-function ((AbM) (9.13.1), (9.13.3), (9.13.7), (LeN) 9.8, 9.13). A characterization of the Dawson function as an sin-integral (over the positive x-axis) of the Gaussian function is given in ((GrI) 3.896 3.). Its Mellin transform is provided in ((GrI) 7.612, (BrK4) lemma S2). The **asymptotics of** the **zeros** of those degenerated hypergeometric functions are given in (SeA) resp. ((BrK4) lemma A4). The **fractional part function** related Zeta function theory is provided in ((TiE) II).

With respect to the considered distributional Hilbert spaces H(-1/2) and H(-1) we note that the Zeta function is an integral function of order 1 and an element of the distributional Hilbert space H(-1). This property is an outcome of the relationship between the Hilbert spaces above, the **Dirichlet series** theory (HaG) and the Hardy space isometry as provided in e.g. ((LaE), §227, Satz 40). With respect to the physical aspects below we refer to (NaS), where the H(1/2) dual space of H(-1/2) on the circle (with its inner product defined by a **Stieltjes integral**) is considered in the context of **Teichmüller theory** and the universal period mapping via **quantum calculus**. For the corresponding Fourier series analysis we refer to ((ZyA) XIII, 11). The approximation by polynomials in a complex domain leads to several notions and theorems of convergence related to **Newton-Gaussian** and **cardinal series**. The latter one are closely connected with certain aspects of the theory of Fourier series and integrals. Under sufficiently strong conditions the cardinal function can be resolved by Fourier's integral. Those conditions can be considerably relaxed by introducing Stieltjes integrals resulting in **(C,1) summable series** ((WhJ1) theorems 16 & 17, (BrK4) remarks 3.6/3.7).

The RH is connected to the **quantum theory** via the Hilbert-Polya conjecture resp. the **Berry-Keating conjecture**. It is about the hypothesis, that the imaginary parts t of the **zeros** 1/2+it of the **Zeta function** Z(t) corresponds to **eigenvalues** of an unbounded self-adjoint operator, which is an appropriate **Hermitian operator** basically defined by QP+PQ, whereby Q denotes the location, and P denotes the (Schrödinger) momentum operator. In (BrK3) the corresponding model (convolution integral) operator S(1) (of order 1 with "density" d(cotx)) for the one-dimensional harmonic quantum oscillator model is provided.

The theory of spectral expansions of non-bounded self-adjoint operator is connected with the notions "Lebesgue-Stieltjes integral" and "**functional Hilbert equation for resolvents** ((LuL) (7.8).The corresponding Hilbert scale framework plays also a key role on the inverse problem for the double layer potential. The corresponding model problem (w/o any compact disturbance operator) with the Newton kernel enjoys a **double layer potential integral operator** with the **eigenvalue 1/2** (EbP).

The Riemann entire Zeta function Z(s) enjoys the functional equation in the form Z(s)=Z(1-s). The alternatively proposed Dawson (baseline) function leads to an alternative entire Zeta function definition Z(*;s) with a corresponding **functional equation** in the form **Z(*,1-s) = Q(s) * Z(*,s)**, with Q(s):=P(s)/P(1-s), whereby P(x):=cx*cot(cx) and the constant c denotes the number "pi"/2. Therefore, the alternative entire Zeta function definition Z(*;s) have same nontrivial zeros as Riemann's entire Riemann Zeta function Z(s).

The RH is equivalent to the **Li criterion** governing a sequence of real constants, that are certain logarithmic derivatives of Z(s) evaluated at unity (LiX). This equivalence results from a necessary and sufficient condition that the logarithmic of the function Z(1/(1-z)) be analytic in the unit disk. The proof of the Li criterion is built on the two representations of the Zeta function, its (product) representation over all its nontrivial zeros ((HdE) 1.10) and Riemann's integral representation derived from the Riemann duality equation, based on the Jacobi theta function ((EdH) 1.8). Based on Riemann's integral representation involving Jacobi's theta function and its derivatives in (BiP) some particular probability laws governing sums of independent exponential variables are considered. In (KeJ) corresponding Li/Keiper constants are considered. The proposed **alternative** entire Zeta function Z(*,s) is suggested to derive an analogue **Li criteron**.

One proof of the Riemann functional equation is based on the **fractional part function** r(x), whereby the zeta function *zeta(s)* in the critical stripe is given by the Mellin transform *zeta*(1-s) = M(-x*d/dx(r(x))(s-1) ((TiE) (2.1.5). The functional equation is given by *zeta*(s) = *chi*(s)**zeta*(1-s), whereby *chi*(s) is defined according to ((TiE) (2.1.12). The Hilbert transform of the fractional part function is given by the log(sin(x))-function. After some calculations (see also (BrK4) lemma 1.4, lemma 3.1 (GrI) 1.441, 3.761 4./9., 8.334, 8.335) the corresponding alternative zeta(*,s) function is given by ***zeta*(*,1-s) * s = *zeta*(1-s) * tan(c*s)**.

The density function J(x) of the log(*zeta*(s)) Fourier inverse integral representation can be reformulated into a representation of the function pi(x) (that is, for the "number of primes counting function" less than any given magnitude x ((EdH) 1.17)). Riemann's proof of the formula for J(x) results into the famous Riemann approximation error function ((HdE) 1.17 (3)) based on the product formula representation of the Gamma function Gamma(1+s/2) ((HdE) 1.3 (4), (GrI) 8.322). The challenge to prove the corresponding **li(x) function** approximation **criterion** (i.e. li(x) - pi(x)=O(log(x)*squar(x)) = O(x*exp(1/2+e)), e>0, (BrK4) p.10) is about the (exponential) asymptotics of the Gaussian function ((EdH) 1.16, (BrK4) note S25). In this context we note that the Dawson function enjoys an only polynomial asymptotics in the form O(x*exp(-1)). In summary, the alternatively proposed Gamma(*,s/2) := Gamma(s/2) * tan(c*s) function leads to an **alternative Riemann approximation error function** with improved convergence behavior (at least with respect to the proposed Hilbert space norms). The appreciated asymptotics of the Dawson function suggested an **alternative li(*,x) function** definition, whereby, of course, the result of Chebyshev about the proven relative error in the approximation of pi(x) by Gauss' li(x) function needs to be taken into account ((EdH) 1.1 (3)). Alternatively to the **Gaussian density** dg=log(1/t)dt the above indicates to consider the **Clausen density** dw, where w(t) denotes the periodical continuation of the Clausen integral ((AbM) 27.8). Obviously the Clausen integral is related to the Hilbert transform of the fractional part function.

The **Dawson function F(x)** (i.e. the Hilbert transform of the Gaussian function f(x):=exp(-(x*x))) is related to the two special Kummer functions K(1,3/2;z) and K(1/2;z):=K(1/2,3/2,z) by **F(x) = x*K(1,3/2;-x*x)** ((LeN) (9.13.3)) resp. **F(x) = x * f(x) * K(1/2,x*x)** ((GrI), 9.212). It provides an option to replace the auxiliary functions G(b) resp. E(b) in (EdH) 1.14, 1.16, to derive the formula for the Riemann density function J(x) ((EdH) 1.12 (2)). Both special Kummer functions enjoy appreciated **non-asymptotics** of its **zeros** (SeA): let c="pi" denote the unit circle constant, then the imaginary part of the zeros of both functions fulfill the inequality **(2n-1)*c<abs(Im(z(n)))<2n*c**, while the real parts fulfill Re(z)<-1/2 resp. Re(z)>1/2 for K(1,3/2;z) resp. K(1/2;z). In other words, there are no zeros of K(1/2;z) on the critical line s=1/2*it (t ex R), resp. there are no zeros of K(1,3/2;z) on the "dual" line (1-s) (see also (BrK4) Notes O5, O22, O23, (BrK7) Note 11).

The density of prime numbers appears to be the **Gaussian density** dg=log(1/t)dt defining the corresponding prime number counting integral function ((EdH) 1.1 (3)). We mention the Kummer function based representation of the li-function in the form li(x)=-x*K(1,1;-logx) ((LeN) (9.13.7)). The asymptotics of the special Kummer functions **K(a;x):=K(a,a+1;x)** are given by K(a;x) ~ e*exp(x+logx) / Gamma(a) ((OlF), 7 §10.1, (AbM) 13.5.1.). Let G(x) denote the first derivative of K(1/2;z), i.e. (d/x) K(1/2;x)=(1/3)*K(3/2;x) with K(3/2;x):=K(3/2,5/2;x), then it holds K(1/2,x)+2xG(x)=e*exp(x) ((BrK4), lemma K2). For the related equations with respect to the incomplete Gamma function we refer to (OlF1) 7.2.2, 8.4.15). The asymptotics of the Kummer functions are given by K(a,c;x) ~ e*exp(x+(a-c)logx) / Gamma(a) ((OlF), 7 §10.1, (AbM) 13.5.1.) Therefore the functions e*exp(x)/x, K(1/2,x) and K(3/2,x) are asymptotically identical. By substitution of the integration variable by t --> exp(y) of the li-function integral this results into an alternative prime number approximation function in the form **K(1/2,logx) = x - logx * (2/3) * K(3/2;logx)**. We also note the relationship of K(a;-x) to the incomplete Gamma function ((AbM) 13.6.10). The **incomplete Gamma function** play a key role to compute the action of the **Leray projection operator** on the **Gaussian functions** (LeN1). Those action formulas can be applied to derive in the context of the **well-posedness** topic of the **NSE** and related (based on tempered distribution and a Carleson measure characterization of the BMO space) estimates ((LeN1), (KoH), theorems 1 and 2, see also (BrK4) pp. 26, 58, 64, 99, 121).

The asymptotics of the special Kummer functions K(a;x):=K(a,a+1;x) ~ e*exp(x+logx) / Gamma(a) ((OlF), 7 §10.1, (AbM) 13.5.1.) is proposed as alternative tool for the additive number theory. **Landau** predicted the proof of the **binary Goldbach conjecture** (with high probability) based on the Stäckel approximation formula in combination with his own corresponding additions (LaE1). With the notation of (LaE1) the prime pair (p,q) counting function H(x) with the condition p+q<= x corresponds asymptotically H(x) ~ (1/2)*(x/logx)*(x/logx). The **Stäckel formula** shows the corresponding asymptotics with respect to the (number theoretical) Euler phi(n)-function in the form (n/logn)*(n/logn)/phi(n). We suggest to apply a modified "density" function in the form **H(*,x) ~ c(a,b) * K(a;logx) * K(b;logx)**. The structure of the alternative prime number approximation function K(1/2,logx) indicates a correspondingly **modified Landau density function** theta(x) = x - c*logx - ... resp. T(x) := theta(e*exp(x)) (as defined and applied e.g. in ((BrK4) pp. 8-10, 23, 104, Notes S29/S30/S56/O51, (KoJ), (LaE) §50), (OsH) Kap. 8)) in the form **theta(*;x)** := K(1/2,logx) - ... = x - logx * (2/3) * K(3/2;logx) - ... .

The relationship of the considered Kummer functions to the incomplete Gamma function is provided in (AbM) 6.5.12. We further note, that the **generalized asymptotic (Poincaré) expansion** admits expansions that have no conceivable value, in an analytical or numerical sense, concerning the functions they represent. In (OlF) §10, the expansion of sin(x)/x is provided with first summand term exp(-x)/logx.

Additionally, the above alternative Z(s) resp. *zeta*(s) function representations indicate an alternative Gamma (auxiliary) function definition in the form G(*,s/2):=G(s/2)*tan(cs)/s with identical asymptotics for x --> 0. Corresponding formulas for the tan(x)- resp. the **log(tan)-function** are provided in ((GrI) 1.421,1.518). In (ElL) the **Fourier expansion** of the log(tan) function is provided, giving a note to its related Hilbert space H(a) regularity. In (ElL1) log-tangent integrals are evaluated by series involving zeta(2n+1).

Its graph looks like a beautiful white noise diagram. In (EsO), formulas (6.3), (6.4), the Fourier expansion of log(Gamma(x)) function is provided with coefficients a(n)=1/(2n), b(n)=(A+logn)/(2cn) and a(0)=log(root(4c)). For a corresponding **Hilbert transform** evaluation we refer to (MaJ).

For other related application areas of Gamma(*,s/2) we refer to Ramanujan's chapter "Analogues of the Gamma Function" ((BeB) chapter 8).

In (TiE) theorem 4.11, an approximation to the *zeta* function series in the critical stripe by a partial sum of its **Dirichlet series** is given ((BrK4) remark 3.8). One proof of this theorem is built on a simple application of the theorem of residues, where the *zeta* series is expressed as a (Mellin transform type) **contour integral** of the **cot(cz)-function** ((TiE) 4.14). As the cot and the *zeta* function are both elements of the distributional Hilbert space H(-1) the contour integral above with a properly chosen contour provides a contour integral representation for the *zeta* in a weak H(-1) sense. In (ChK) VI, §2, two expansions of cot(z) are compared to prove that all coefficients of one of this expansion (zeta(2n)/pi(exp(2n))) are rational. Corresponding formulas for odd inters are unknown. In (EsR), example 78, a **"finite part"-"principle value" integral representation** of the **c*cot(cx)** is given (which is zero also for positive or negative integers). It is used as enabler to obtain the asymptotic expansion of the p.v. integral, defined by the "restricted" Hilbert transform integral of a function u(x) over the positive x-axis, only. In case u(x) has a structure u(x)=v(x)*squar(x) the representation enjoys a remarkable form, where the numbers n+1/2 play a key role. In (OlF1) 25.6.6, an integral value representation for zeta(2n+1) is provided with cot(2cx) "density" function.

In ((BrK4) lemma 3.4, lemma A12/19) the function P(x) is considered in the context of (appreciated) quasi-asymptotics of (corresponding) distributions ((ViV) p. 56/57) and the Riemann mapping theorem resp. the Schwarz lemma. The considered "function" g(x):=-d/dx(cot(x)) (whereby the cot-"function" is an element of H(-1)) is auto-model (or regular varying) of order -1. This condition and its corresponding asymptotics property ((BrK) lemma 3.4) provide the prereqisitions of the **RH Polya criterion** ((PoG), (BrK5) theorem 6). The above quasi-asymptotics indicates a replacement of the differential d(logx)by d(log(sinx)). The cot(z) function expansions (ChK) VI, §2) in combination with **Ramanujan's formula** ((EdH) 10.10) resp. its generalization theorem ((EdH) p.220) is proposed to be applied to define an appropriate analytical (Mellin transform) function in the stripe 1/2<Re(s)<1.

In (GrI) 8.334, the relationship between the the cot- and the Gamma function is provided. From (BeB) 8. Entry17(iii)) we quote: "*the* **indefinite *Fourier series of the cot(cx)-function*** *may be formally established by differentiating the corresponding Fourier series equation for* (the L(2)=H(0)-function) ***-log(2sin(cx))***" ((BrK4) remark 3.8). The proposed distributional Hilbert scales provide the proper framework to justify Ramanujan's related parenthetical remark "*for the same limit*" (in a H(-1)-sense).

The related NSE, YME and geometrodynamics solution concept(s)

The common Hilbert scale is about the Hilbert spaces H(a) with a=1,1/2,0,-1/2,-1 with its corresponding inner products **((**u,v**))**, **(**u,v**)**, (u,v), ((u,v)), (((u,v))). The proposed mathematical concepts and tools are especially correlated to the names of **Plemelj**, **Stieltjes** and **Calderón**.

The **Dirac theory** with its underlying concept of a "Dirac function" (where the regularity of the Dirac distribution "function" depends from the space dimension) is **omitted** and replaced by a distributional Hilbert space (domain) concept. This alternative concept avoids space dimension depending regularity assumptions for (quantum) physical variational model (wave package) states and solutions (defined e.g. by energy or operator norm minimization problems) and physical problem specific "force" types. We note that for signals on R the spectrum of the Hilbert transform is (up to a constant) given by the distribution v.p.(1/x), whereby the symbol "v.p." denotes the Cauchy principal value of the integral over R. Its corresponding Fourier series is given by -i*sgn(k) with its relationship to "positive" and "negative" Dirac "functions" and the unit step function Y(x). In a H(-1/2) framework the Dirac "function" concept can be avoided, which enables a generalization to dimensions n>1 without any corresponding additional regularity requirements (the **Dirac/Delta** "**function**" is **an element of H(-n/2-e)**, e>0).

The newly proposed "fluid/quantum state" Hilbert space H(-1/2) with its closed orthogonal subspace of H(0) goes also along with a combined usage of L(2) waves governing the H(0) Hilbert space and "orthogonal" wavelets governing the H(-1/2)-H(0) space. The wavelet "reproducing" ("duality") formula provides an additional degree of freedom to apply wavelet analysis with appropriately (problem specific) defined wavelets, where the "*microscope observations*" of ***two*** **wavelet (optics) functions** can be compared with each other (LoA). The prize to be paid is about additional efforts, when re-building the reconstruction wavelet.

We propose modified Maxwell equations with correspondingly extended domains according to the above. This model is proposed as alternative to SMEP, i.e. the modified Maxwell equation are proposed to be a "Non-standard Model of Elementary Particles (NMEP)", i.e. an alternative to the Yang-Mills (field) equations. The conceptual approach is also applicable for the Einstein field equations. Mathematical speaking this is about potential functions built on correponding "density" functions. The source density is the most prominent one. Physical speaking the source is the root cause of the corresponding source field. Another example is the invertebrate density (=rotation) with its corresponding rotation field. The Poincare lemma in a 3-D framework states that source fields are rotation-free and rotation fields are source-free. The physical interpretation of the rotation field in the modified Maxwell equations is about rotating "mass elements w/o mass" (in the sense of Plemelj) with corresponding potential function. In a certain sense this concept can be seen as a generalization of the Helmholtz decomposition (which is about a representation of a vector field as a sum of an irrotational (curl-free) and a solenoidal (divergence-free) vector field): it is derived applying the delta "function" concept. In the context of the proposed distributional Hilbert space framework, the Dirac function concept (where the regularity of those "function" depends from the space dimension) is replaced by the quantum state Hilbert space H(-1/2). The solution u (ex H(1/2)) of the Helmholtz equation in terms of the double layer potential is provided in ((LiI), 7.3.4). From the Sobolev embedding theorem it follows, that for any space dimension n>0 the modified Helmholtz equation is valid for not continuous vector fields.

A Kummer function based RH solution concept(s)

The **RH** is connected to the **quantum theory** via the Hilbert-Polya conjecture resp. the **Berry-Keating conjecture**. The latter one is about a physical reason, why the RH should be true. This would be the case if the imaginary parts t of the zeros 1/2+it of the **Zeta function** Z(t) corresponds to eigenvalues of an unbounded self-adjopint operator, which is an appropriate **Hermitian operator** basically defined by QP+PQ, whereby Q denotes the location, and P denotes the (Schrödinger) momentum operator. The notion "***unbounded***" is not well defined, as an operator is only well-defined by describing the operator "mapping" in combination with its defined domain. The Zeta function is an **element of H(-1)**, but not an element of H(-1/2).Therefore, there is a characterization of the Zeta function on the critcal line in the form ((Z,v)) for all v ex H(0). As the "test space" H(0) is compactly embedded into H(-1/2) this shows that there is an extended Zeta function Z(*)=Z+Z(#) (Friedrichs' extension) with the characterization ((Z(*),v)) for all v ex H(-1/2), where Z can be interpreted as orthogonal approximation of Z(*) with discrete spectrum.

Riemann's "workaround" function h(x):-x*d/dx(f(x) do have an obvious linkage to the "**commutator**" concept in quantum theory. In this context the Gaussian function f(x) can be characterized as "minimal function" for the Heissenberg uncertainty inequality. Applying the same solution concept as above then leads to an alternative Hilbert operator based representation in H(-1/2), resp. to a H(-1) based definition of the commutator operator with extended domain. The common denominator of the alternatively proposed Hilbert space framework H(-1/2) goes along with the definition of a correspondingly defined "momentum" operator (of order 1) P: H(1/2) --> H(-1/2) defined in a variational form. In the one-dimensional case the Hilbert transform H (in the n>1 case the **Riesz operators R**) is linked to such an operator given by ((Pu,v))=(Hu,v). With respect to quantum theory this indicates an **alternative Schrödinger momentum operator** (where the gradient operator "grad" is basically replaced by the Hilbert transformed gradient, i.e. **P**:=-i***R**(grad) and a corresponding alternative commutator representation Q**P**-**P**Q in a weak H(-1/2) form. We note that the **Riesz operators R** commute with **translations** and **homothesis** and enjoy nice properties relative to **rotations**.

The related NSE problem/solution area

The **NSE** are derived from the (Cauchy) stress tensor (resp. the shear viscosity tensor) leading to liquid pressure force. In electrodynamics& kinetic plasma physics the linear resp. the angular momentum laws are linked to the electrostatic (mass “particles”, collision, static, quantum mechanics, displacement related; “fermions”) Coulomb potential resp. to the magnetic (mass-less “particles”, collision-less, dynamic, quantum dynamics, rotation related; “bosons”) Lorentz potential.

With respect to the open Millenium **3D non-stationary, non-linear NSE** problem we note that the alternatively proposed **"fluid state" Hilbert space H(-1/2)** with corresponding alternative energy ("**velocity**") space H(1/2) enables a (currently missing) energy inequality based on existing contribution of the non-linear term. In the standard weak NSE representation this term is zero, which is a great thing from a mathematical perspective, avoiding sohisticated estimating techniques, but a doubtful thing from a physical modelling perspective, as this term is the critical one, which jepordized all attempts to extend the 3D problem based on existing results from the 2D case into the 3D case. The corresponding estimates are based on Sobolev embedding theorems; the **Sobolevskii estimate** provides the appropriate estimate given that the "fluid state" space is H(-1/2) in a corresponding weak variational representation.

A "3D challenge" like the NSE above is also valid, when solving the monochromatic scattering problem on surfaces of arbitrary shape applying electric field integral equations. From (IvV) we recall that the (integral) operators A and A(t): H(-1/2) --> H(1/2) are bounded Fredholm operators with index zero. The underlying framework is still the standard one, as the domains are surfaces, only. An analog approach as above with correspondingly defined surface domain regularity is proposed.

The related YME problem/solution area

With respect to the YME the proposed mathematical concepts and tools are especially correlated to the names of **Schrödinger** and **Weyl** (e.g. in the context of "half-odd integers quantum numbers for the Bose statistics" and resp. Weyl's contributions on the concepts of matter, the structure of the world and the principle of action (WeH), (WeH1), (WeH2)). It enables an **alternative** (quantum) **ground state energy model** embedded in the proposed distributional Hilbert scale frame of this homepage covering all variational physical-mathematical PDE and Pseudo Differential Operator (PDO) equations (e.g. also the Maxwell equations).

The electromagnetic interaction has gauge invariance for the probability density and for the Dirac equation. The wave equation for the gauge bosons, i.e. the **generalization** of the **Maxwell equations**, can be derived by forming a gauge-invariant field tensor using generalized derivative. There is a parallel to the definition of the covariant derivative in general relativity. With respect to the above there is an alternative approach indicated, where the fermions are modelled as elements of the Hilbert space H(0), while the complementary closed subspace H(-1/2)-H(0) is a model for the "**interaction particles, bosons**". For gauge symmetries the fundamental equations are symmetric, but e.g. the ground state wave function breaks the symmetry. When a gauge symmetry is broken the gauge bosons are able to acquire an effective mass, even though gauge symmetry does not allow a boson mass in the fundamental equations. Following the above alternative concept the "symmetry state space" is modelled by H(0), while the the ground state wave function is an element of the closed subspace H(-1/2)-H(0) of H(-1/2) (BrK).

When one wants to treat the time-harmonic Maxwell equations with variational methods, one has to face the problem that the natural bilinear form is not coercive on the whole Sobolev space. On can, however, make it coercive by adding a certain bilinear form on the boundary of the domain (vanishing on a subspace of H(1)), which causes a change in the natural boundary conditions.

In SMEP (**Standard Model of Elementary Particles**) symmetry plays a key role. Conceptually, the SMEP starts with a set of fermions (e.g. the electron in quantum electrodynamics). If a theory is invariant under transformations by a symmetry group one obtains a conservation law and quantum numbers. Gauge symmetries are local symmetries that act differently at each space-time point. They automatically determine the interaction between particles by introducing bosons that mediate the interaction. U(1) (where probability of the wave function (i.e. the complex unit circle numbers) is conserved) describes the elctromagnetic interaction with 1 boson (photon) and 1 quantum number (charge Q). The group SU(2) of complex, unitary (2x2) matrices with determinant I describes the weak force interaction with 3 bosons (W(+), W(-), Z), while the group SU(3) of complex, unitary (3x3) matrices describes the strong force interaction with 8 gluon bosons.

Maxwell, Young-Mills and gravitation field equations

Reformulated Maxwell or gravitation field equations in a weak H(-1/2)-sense leads to the same effect, as dealing with an isometric mapping g --> H(g) in a weak H(0)-sense (H denotes the Hilbert transform) alternatively to a second order operator in the form x*P(g(x)) in a weak H(-1/2). This results into some opportunities as

- the solutions of the Maxwell equations in a vaccum do not need any callibration transforms to ensure wave equation character; therefore, the arbitrarily chosen Lorentz condition for the electromagnetic potential (to ensure Lorentz invariance in wave equations) and its corresponding scalar function ((FeR), 7th lecture) can be avoided

- enabling alternative concepts in GRT to e.g. current (flexible") metrical affinity, affine connexions and local isometric 3D unit spheres dealing with rigid infinitesimal pieces, being replaced by **geometrical manifolds**, enabling isometrical stitching of rigid infinitesimal pieces ((CiI), (ScP)).

The related geometrodynamics problem/solution area

Replacing the affine connexion and the underlying covariant derivative concept by a geometric structure with corresponding inner product puts the spot on the

Thurston conjecture*: The interior of every compact 3-manifold has a canonical decomposition into pieces which have geometric structure (ThW).*

This conjecture asserts that any compact 3-manifold can be cut in a reasonably canonical way into a union of geometric pieces. In fact, the decomposition does exist. The point of the conjecture is that the pieces should all be geometric. There are precisely eight homogeneous spaces (X, G) which are needed for geometric structures on 3-manifolds. The symmetry group SU(2) of quaternions of absolute value one (the model for the weak nuclear force interaction between an electron and a neutrino) is diffeomorph to S3, the unit sphere in R(4). The latter one is one of the eight geometric manifolds above (ScP). We mention the two other relevant geometries, the Euclidean space E3 and the hyperbolic space H3. It might be that our universe is not an either... or ..., but a combined one, where then the "connection" dots would become some physical interpretation. Looking from an Einstein field equation perspective the Ricci tensor is a second order tensor, which is very much linked to the Poincare conjecture, its solution by Perelman and to S3 (AnM). The **geometrodynamics** provides alternative (pseudo) tensor operators to the Weyl tensor related to H3 (CiI). In (CaJ) the concept of a Ricci potential is provided in the context of the Ricci curvature equation with rotational symmetry. The single scalar equation for the Ricci potential is equivalent to the original Ricci system in the rotationally symmetric case when the Ricci candidate is nonsingular. For an overview of the Ricci flow regarding e.g. entropy formula, finite extinction time for solutions on certain 3-manifolds in the context of Prelman's proof of the Poincare conjecture we refer to (KlB), (MoJ).

The single scalar equation for the Ricci potential (CaJ) might be interpreted as the counterpart of the **CLM vorticity equation** as a simple one-dimensional turbulent flow model in the context of the NSE.

The link back to a Hilbert space based theory might be provided by the theory of spaces with an indefinite metric ((DrM), (AzT), (DrM), (VaM)). In case of the L(2) Hilbert space H, this is about a decomposition of H into an orthonal sum of two spaces H1 and H2 with corresponding projection operators P1 and P2 relates to the concepts which appear in the problem of S. L. Sobolev concerning **Hermitean operators** in spaces with **indefinite metric** ((VaM) IV). For x being an element of H this is about a defined "**potential**" p(x):=<<x>>*<<x>> ((VaM) (11.1)) and a corresponding **"grad"** potential operator **W**(x), given by

grad(p(x)):=2**W**(x):=P1(x)-P2(x)   (VaM) (11.4).

The potential criterion p(x)=c>0 defines a manifold, which represents a **hyperboloid in the Hilbert space H** with corresponding hyperbolic and conical regions. The tool set for an appropropriate **generalization** of the above "grad" definition is about the (homogeneous, not alway non-linear in h) **Gateaux differential** (or weak differential) **VF**(x,h) of a functional F at a point x in the direction h ((VaM) §3)). The appropriate weak inner product might be the inner product of the "velocity" space H(1/2). We note the Sobolev embedding theorem, i.e. H(k) is a sub-space of C(0) (continuous functions) for k>n/2, i.e. there is no concept of "continuous velocity/momentum" in the proposed Hilbert space framework, i.e. there is no Frechet differential existing ((VaM) 3.3). This refers to one of the several proposals, which have been made to drop some of the common sense notions about the universe ((KaM) 1.1), which is about continuity, i.e. space-time must be granular. The size of these grains would provide a natural cutoff for the Feynman integrals, allowing to have a finite S-matrix.

A selfadjoint operator B defined on all of the Hilbert space H is bounded. Thus, the operator B induces a decomposition of H into the direct sum of the subspaces, and therefore generates related hyperboloids ((VaM) 11.2). Following the investigations of Pontrjagin and Iohvidov on linear operators in a Hilbert space with an indefinite inner product, M. G. Krein proved the **Pontrjagin-Iohvidov-Krein theorem** (FaK).

In an universe model with appropriately connected geometric manifolds the corresponding symmetries breakdowns at those "connection dots" would govern corresponding different conservation laws in both of the two connected manifolds. The Noether theorem provides the corresponding mathematical concept (symmetry --> conservation laws; energy conservation in GT, symmetries in particle physics, global and gauge symmetries, exact and broken). Those symmetries are associated with "non-observables". Currently applied symmetries are described by finite- (rotation group, Lorentz group, ...) and by infinite-dimensional (gauged U(1), gauged SU(3), diffeomorphisms of GR, general coordinate invariance...) Lie groups.

A manifold geometry is defined as a pair (X,G), where X is a manifold and G acts transitively on X with compact point stabilisers (ScP). Related to the key tool "Hilbert transform" resp. "conjugate functions" of this page we recall from (ScP), that Kulkarni (unpublished) has carried out a finer classification in which one considers pairs (G,H) where G is a Lie group, H is a compact subgroup and G/H is a simple connected 3-manifold and pairs (G1,H1) and (G2,H2) are equivalent if there is an isomorphism G1 --> G2 sending H1 to a conjugate of H2. Thus for example, the geometry S3 arises from three distinct such pairs, (S3,e), (U(2),SO(2)), (SO(4),SO(3)). Another example is given by the Bianchi classification consisting of all simply connected 3-dimensional Lie groups up to an isomorphism.

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