The Gravity and Quantum Field Model

A „Form“–„Fit“–„Function“ validation approach across affected paradigms

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Mathematics is a formal science. It is neither part of liberal arts nor of natural sciences. Exact sciences are part of natural sciences targeting for mathematically described theories free from natural languages. Natural sciences are concerned with descriptions and understanding of natural phenomena governed by empirical evidence.

The purpose of this note is to provide a kind of consistency check of the proposed Hilbert space based gravity and quantum field model, which is basically a (coarse-grained) Minkowski space-time continuum compactly embedded into an (Hilbert-) Minkowski „ether“ continuum, (EiA2). The methodology for this kind of consistency check is about the „Form – Fit - Function“ concept.

The „Form“ in the context of this note is given by the proposed gravity and quantum field model providing an overall „language“ standard. (*).

The Fit & Function of the „Form“ is about consistent mappings to related notions of affected paradigms in related exact sciences or philosophy areas and resolving corresponding current intra- or inter-paradigms challenges.

Regarding the conceptual differences between „Quantum Mechanics versus General Relativity“ the identifcation of the affected physical paradigms of the proposed gravity and quantum field model is guided by the model problem, „The Problem of Time,“ (**).

Regarding physical aspects of living cells and philosophical views of the world the identification of affected paradigms is guided by Schrödinger‘s view of the world, (ScE). It contains notions like „mind, matter, living cells, consciousness, organic, inorganic, mneme“ associated with the names of Plato, Kant, Schopenhauer, and Einstein (ScE1) (***), (ScE2).

(*) Wittgenstein L., (WiL): „The book deals with the problems of philosophy and shows, as I believe, that the method of formulating these problems rests on the missunderstanding of the logic of our language. Its whole meaning could be summed up somewhat as follows: What can be said at all can be said clearly; and whereof one cannot speak thereof one must be silent.

The book will, therefore, draw a limit to thinking, but to the expression of thoughts; for, in order to draw a limit to thinking we should have to be able to think both sides of this limit (we should therefore have to be able to think what cannot be thought).“

(**) (AnE): „In approaching Quantum Gravity, many conceptual issues turn out to be related to notions of time. This occurs because notions of time are substantially different across the Newtonian Physics, Special Relativity (SR), Quantum Mechanics (QM), Quantum Field Theory (QFT) and General Relativity (GR). A first example in which this occurs is QM versus GR. …. The problem of time is, in greater generality, a consequence of the missmatch between Background Dependent and Background Independent Paradigms of Physics. Newtonian Physics, SR, QM, and QFT are all Background Dependent, whereas GR is Background Independent and many approaches to Quantum Gravity expect this to be Background Independent as well.“

(***) (ScE1) p.141: „I say, the most appreciable help science has offered us (in this) is, in my view, the gradual idealization of time. In thinking of this the names of three men obtrude themselves upon us, though many others, including non-scientists, have hit on the same groove, such as St Augustine of Hippo and Boethius; the three are Plato, Kant and Einstein. … In Plato‘s case it came from mathematics and geometry. … In my opinion it was this, that he was the first to evisage the idea of timeless existence and to enphasize it – against reason – as a reality, more real than our actual experience; this he said, is but a shadow of the former, from which all experienced reality is borrwed. I am speaking of the theory of forms (or ideas).

Penrose’s speculation about time (1989)
(PeR1) p.443-4: „I suggest that we may actually be going badly wrong when we apply the usual physical rules for time when we consider consciousness! . . . My guess is that there is something illusory here. . .and the time of our perceptions does not 'really' flow in quite the linear forward-moving way that we perceive it to flow (whatever that might mean!). The temporal ordering that we 'appear' to perceive is, I am claiming, something that we impose upon our perceptions in order to make sense of them in relation to the uniform forward time-progression of an external physical reality.“

Below there are three diagrams providing the conceptual element of the considered mathematical, physical and philosophical paradigms.

There is an 100% - fit to the Quantum Mechanics and and the Special Relativity Theory with its underlying Minkowski space. The „give aways“ are

i) Riemann's continuous manifold concept

ii) the last mechanical quality, namely, the immobility of the Lorentz transformation

Riemann's used the notion „continuous manifold“ synonymonly for the notion "multiple extended quantities" (*). We note that in order to be applicable to the GRT it must be extented to „differentiable manifolds“, just for mathematical reasons w/o any physical meaning.

There is a 0% - fit to the quantum field theory (the SMEP model with its glued together symmetry groups $U(1)xSU(2)xSU(3)$), the loop quantum gravity, (RoC), and the (super) string theory (M-Theory), (KaM). One consequence out of this is the disappearance of the mass gap problem of the Yang-Mills Equations.

Depending from the interpretation (or more precisely, depending from the definition) of the philosophical notions of Kant and Schopenhauer, there is also a 100% - fit to their concepts.

As a side note we remark, that Heidegger's „Being & Time“ is only about „Being“. The mathematical model might provide some arguments, why the famous book of M. Heidegger never has been completed, as announced in (HeM) §8 „*The outline of the treatise*“ (*).

(*) *Dedekind R., (DeR): „If space has at all a real existence it is not necessary for it to be continuous; many of its properties would remain the same even were discontinuous. And if we knew for certain that space was discontinuous there would be nothing to prevent us, in case we so desire, from filling up ist gaps, in thought, and thus making it continuous; this filling up would consist in a creation of new point-individuals and would have to be effected in accordance with the above principle."*

(**) *§ 8: Die Ausarbeitung der Seinsfrage gabelt sich so in zwei Aufgaben; ihnen entspricht die Gliederung der Abhandlung in zwei Teile:*
Erster Teil: Die Interpretation des Daseins auf die Zeitlichkeit und Explikation der Zeit als des transzendentalen Horizontes der Frage nach dem Sein.
Zweiter Teil: Grundzüge einer phänomenologischen Destruktion der Geschichte der Ontologie am Leitfaden der Problematik der Temporalität.
Der erste Teil zerfällt in drei Abschnitte:

1. *Die vorbereitende Fundamentalanalyse des Daseins.*
2. *Dasein und Zeitlichkeit.*
3. *Zeit und Sein.* (never published)

*Der Zweite Teil (*nothing published in the book*) gliedert sich dreifach:*

1. *Kants Lehre vom Schematismus und der Zeit als Vorstufe einer Problematik der Temporalität*
2. *Das ontologische Fundament des „cogito ergo sum" Descartes' und die Übernahme der mittelalterlichen Ontologie in die Problematik der „res cogitans".*
3. *Die Abhandlung des Aristoteles über die Zeit als Diskrimen der phänomenalen Basis und der Grenzen der antiken Ontologie.*

The Gravity and Quantum Field Model
The Form

Regarding the notion „form“ we recall from E. Schrödinger, Science and Humanism, (ScE2) p. 122:

> *Form, not Substance, the Fundamental Concept*
>
> *The situation is rather disconcerting. You will ask: What are these particles then, if they are not individuals? And you may point out another kind of gradual transition, namely that between an ultimate particle and a palpable body in our environment, to which we do attribute individual sameness. A number of particles consitute a atom. Several atoms constitute a molecule. Molecules there are of various sizes, small ones and big ones, but without their being any limit beyond which we call it a big molecule. In fact there is no upper limit to the size of a molecule, it may contain hundreds of thousands of atoms. It may be a virus or a gene, vissible under the microscope. Finally we may observe that any palpable object in our environment is composed of molecules, which ae composed of atoms, which are lack individuality, how does, say, my wrist-watch come by individuality? Where is the limit? How does individuality arise at all in objects composed of non-individuals? ….*
>
> *Let us now return to our ultimate particles and to small organizations of particles as atoms or small molecules. The old idea about them was that their individuality was based on the identity of matter in them. This seems to be a gratuitous and almost mystical addition that is in sharp contrast to what we have just found to constitute the individuality of macroscopic bodies, which is quite independent of such crude materialistic hypothesis and does not need its support. The new idea is that what is permanent in these ultimate particles or small aggregates is their shape and organization (the form). The habit of everyday language deceives us and seems to require, whenever we hear the word „shape“ or „form“ pronounced, that it must be the shape or form of something. That a material substratum is required to take on a shape. Scientfically this goes back to Aristotle, his causa materialis and causa formalis. But when you come to the ultimate particles constituting matter, there seems to be no point in thinking of them again as constituting of some material. They are, as it were, pure shape (or form) nothing but shape/form; what turns up again and again in successive observations is the form, not an individual speck of material.“*

The central concept for statistical analysis of empirical data in exact sciences is the L_2 Hilbert space equipped with the inner product $(u,v)_{L_2}$ of Lebesgue square-integrable functions. The concept of a Hilbert space generalizes the notion of Euclidean space. The most famous example came along with the Special Relativity Theory (SRT); it is the change from the Euclidean space to the Minkowski space.

Each Hilbert space is a metric space (the metric is given by the norm defined by the inner product), but not the other way around. The usual example to show this, is the metric $\mathrm{d}(x,y)=\sum_{n=1}^{\infty}2^{-n}\frac{|x_n-y_n|}{1+|x_n-y_n|}$, where x_n, y_n are sequences of real numbers, which is not a norm, i.e. it cannot be built by an inner product. A complete normed space is called a Banach space, e.g. R^n equipped with the norm $\|x\|_p^p=\sum_{n=1}^{\infty}|x_n|^p$ is a Banach space.

The inner product of a Hilbert space is its characterizing operations, which allows to define not only lengths but also angles, i.e. it makes a Hilbert space a „geometrical framework“, in contrast to purely metric spaces having no geometric structure at all.

The L_2 Hilbert space is the central concept in thermostatistics and quantum mechanics. Its essential properties in those areas are

i) $L_2=H_0$ is reflexive with respect to the inner product $(u,v)_{L_2}$, i.e. $H_0=H_0^*$,

ii) $L_2=H_0$ is separable, which means that its dimension is countable, i.e. the dimension is the smallest possible infinite dimension $\aleph$.

Those properties are important in the context of „observables“ modelled as linear operators with defined domain sub-spaces of H_0 and their underlying eigenpair solutions with their relationship to the Heisenberg uncertainty inequality (which is nothing else than a formula about statistical variances).

The countable set of eigenpair solutions allows the building of reflexive Hilbert scales $H_\alpha, \alpha\in R$ in the form $H_\alpha=H_{-\alpha}^*$ with respect to the inner product $(u,v)_{L_2}$. The central property in the context of the proposed gravity and quantum field model is the fact, that there is a compact embeddedness of H_α into H_β for every $\alpha>\beta$ and the closure of H_α with respect to the norm of H_β gives the Hilbert space H_β. This densely embeddingness corresponds to the rational numbers as countable sub-set of the field of real numbers.

The Hilbert scale model enables a coarse-grained Hilbet space as a sub-Hilbert-space of an overall Hilbert space addressing the following modelling „requirements/challenges“:

> (WeH) p. 171: „*On the basis of rather convincing general considerations, G. Mie in 1912 pointed out a way of modifying the Maxwell equations in such a manner that they might possibly solve the problem of matter, by explaining why the field possesses a granular structure and why the knots of energy remain intact in spite of the back-and-forth flux of energy and momentum*“.
>
> (FeE:„*Dirac's theory of radiation is based on a very simple idea; instead of considering an atom and the radiation field with which it interacts as two distinct systems, he treats them as a single system whose energy is the sum of three terms: one representing the energy of the atom, a second representating the electromagnetic energy of the radiation field, and a small term representing the coupling energy of the atom and the radiation field*.“

The choice of $\beta := -1/2 < 0$ is basically motivated by the Sobolev embedding theorem in the context of Dirac's physical „point charge“ model of the „ideal (Dirac) function“ $\delta \in \mathrm{H}_{-n/2-\varepsilon}$, which is proposed to be replaced by quantum elements $\in \mathrm{H}_{-1/2} = L_2 \otimes L_2^{\perp}$.

Correspondingly, the (thermo-) statistics Hilbert space L_2 is extended to $H_{-1/2} = L_2 \otimes L_2^{\perp} = H_0 \otimes H_0^{\perp}$, and standard PDE variational representations in the form $u \in D_0(\mathrm{B})\colon (Bu,v)_{0,}\ \forall\, v \in H_0$, are considered as approximations to extended (weak) variational representations in the form

$$u \in \mathrm{D(B)}\colon (Bu,v)_{-1/2,}\ \ \forall\, v \in H_{-1/2}.$$

The considered decompositions $\mathrm{H}_{-1/2} = L_2 \otimes L_2^{\perp}$ resp. $\mathrm{H}_{1/2} = H_1 \otimes H_1^{\perp}$ are about

- a „coarse-grained“ (discrete spectrum/orthogonal eigenfunctions based) Hilbert space L_2 resp. H_1; they represent the scope of exact sciences, e.g. equipped with the notions „time“, kinematical energy“,, „action“, „causality“, „wave equation“

- a closed sub-spaces $L_2^{\perp}$ resp. $H_1^{\perp}$ (e.g. modelling wave packages) of $\mathrm{H}_{-1/2}$ resp. $\mathrm{H}_{1/2}$. The dimension of the Hilbert space L_2 is $\aleph$, in opposite to the much more larger „size“ of $L_2^{\perp}$ and $\mathrm{H}_{-1/2}$ is $2^{\aleph}$.

The model also allows the definition of a potential operator $\boldsymbol{W}(x)$ in the form, (VaM) (11.4)

$$\boldsymbol{W}(x) := \frac{1}{2}\mathrm{grad}(\varphi(x)) := P^1(x) - P^2(x)$$

and a corresponding potential criterion $\varphi(x) = c > 0$ to extend the concept of kinematical energy to non-kinematical energy. We emphasis that the concept of potential energy becomes now an intrinsic part of the mathematical world, independently from considerered physical phenomena (i.e. PDE). In physics the „*potential*“ is defined as the ratio of $\frac{E_{pot}}{c_{coupl}}$, where the coupling constant c_{coupl} depends from the considered physical problem.

The essential new element of the proposed quantum field model is the concept of a „*quantum element*“, which carries both, kinematical and potential energy, in the following form

$$|||x|||_E^2 = |||x|||_{E_{kin}}^2 + |||x|||_{E_{pot}}^2.$$

In case two quantum elements have the same kinetimatical energy, but different potential energy, there is a potential energy difference, which interacts with the common kinematical part of the two quantum elements. In the proposed model the Hilbert space $H_{EP_{kin}}$ of „kinematical quantum elements“ (~ „fermions“) is compactly embedded into the overall Hilbert space of quantum elements H_{EP}, i.e., mathematically speaking, the cardinality of the space $H_{EP_{kin}}$ is identical with the cardinality of the rational numbers, which is $\aleph$, while the cardinality of the overall quantum element Hilbert space is identical with the cardinality of the real numbers, which is $2^{\aleph}$. This properties enable corresponding discrete and continuous spectra. Physically speaking, purely potential quantum elements correspond to the concept of wave packets governed by wavelets (DeL), (HoM), while purely kinematical quantum elements correspond to the concept of Fourier waves.

With respect to the below diagrams we formulate a boundary value model problem from elasticy theory, which is concerned with weak variational representation of tensor field based PDE, (VeW), see also (ArA):

$$-s_{jk,k} = f_j \text{ in } G \text{ , } u_j = u_j^0 \text{ on } \partial G.$$

The vector $f = (f_1, f_2, f_3)$ defines the (external) volume forces which act on the body.

The stress tensor s_{jk} is coupled with the strain tensor $e_{jk} := \frac{1}{2}(u_{j,k} + u_{k,j})$ by the Hook law in the form

$$s_{jk} \coloneqq c_{jkmn} e_{mn}.$$

In this case the c_{jkmn} are constants, but they can be extended to a tensor field.

In case the coefficients are symmetric, i.e. $c_{jkmn} = c_{kjmn} = c_{mnjk}$ it holds $s_{jk} = s_{kj}$.

Given that the elasticy potential

$$W(e) := \frac{1}{2} c_{jkmn} e_{jk} e_{mn}$$

is a positive definite, quadratic form, i.e. $W(e) \geq c e_{jk} e_{jk}$ for $e_{jk} = e_{kj}$ the double potential energy of a displacement $u = (u_1, u_2, u_3)$ is given by

$$(*) \qquad J(u) = a(u,u) - l(u)$$

With

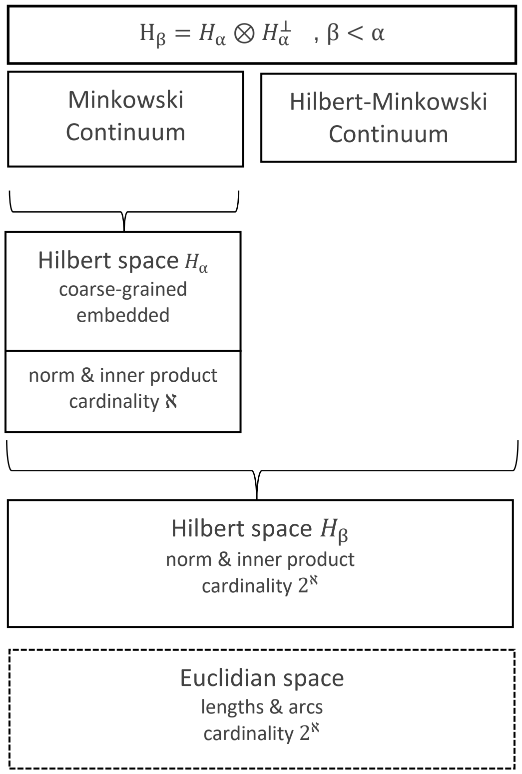
$$a(u,v) := \frac{1}{4} \int_G c_{jkmn}(u_{j,k} + u_{k,j})(v_{j,k} + v_{k,j})dx \text{ , } l(u) := \int_G f_j u_j dx \text{ , } f_j \in L_2(G).$$

Then, the principle of the minimum of potential energy for a given $u_0 = (u_1^0, u_2^0, u_3^0)$, $u_j^0 \in H_1(G)$, is given by the equivalent representations

$$J(u) \to min \text{ , } \quad u - u_0 \in (H_0^1(G))^3 \quad \text{iff} \quad a(u,v) = l(v) \quad \forall \, v \in (H_0^1(G))^3.$$

The Gravity and Quantum Field Model

The Mathematical Model Diagram



The Gravity and Quantum Field Model

The Physical Model Diagram

$H_\beta = H_\alpha \otimes H_\alpha^\perp$, $\beta = \pm 1/2 < \alpha = \pm 1$

Lorentz Space-Time

Einstein Ether

PDE variational representations in the form
$u \in D_0(B)$:
$(Bu, v)_{0,}\ \forall\, v \in H_0$

Hilbert space H_0
coarse-grained embedded into
$H_{-1/2} = H_0 \otimes H_0^\perp$

governed by the principle of action minimization and the paradigm of thermostatistics

PDE variational representations in the form
$u \in \overline{D(B)}$:
$(Bu, v)_0 \sim (u, v)_1\ \forall v \in H_0$

considered as weak approximation representations to ....

... $u \in \overline{D(B)}$: $(Bu, v)_{-1/2} \sim (u, v)_{1/2}\quad \forall\, v \in H_{1/2}$.

governed by the principle of
energy minimization
e.g. „explaining“ the entropy phenomenon,
formulated as the 2nd law of thermodynamics in H_0

The Gravity and Quantum Field Model

The Philosophical Model Diagram

$$H_\beta = H_\alpha \otimes H_\alpha^\perp \quad , \beta = \pm 1/2 < \alpha = \pm 1$$

Space-Time Continuum	Ether Continuum

Schopenhauer
The World as Will and Representation

Representation	Will
appearance; empirical consciousness & apperance background; better consciousness (the true essense of empirical appearance)	universal & irrational force, which acts everywhere and immutable and which stands outside of space and time

Why?	What?
On the fourfold root of the principle of sufficient reason Grund des Werdens (kausal) Grund des Erkennens (Logik) Seinsgrund (Raum-Zeit) Motiv (Handeln)	The Thing in Itself
Maya	Brahman

Kant
The Critique of Pure Reason

Categories & Intuitive Forms	The Thing in Itself

Plato
Politeia

Experience (appearance, dóxa)	The Idea (the Being, ón)

Continuum

Some quotes from E. Schrödinger and H. Weyl:

(ScE2) p. 133: The Intricacy of the continuum

However painful its loss may be, by losing it we probably lose something that is very well worth losing, It seems simple to us, because the idea oft he continuum seems simple to us. We have somehow lost sight of the difficulties it implies. That is due to a suitable conditioning in early childhood. Such an idea as „all numbers between 0 and 1" or „all the numbers between 1 and 2" (*), has become quite familar to us. We just think of them geometically as the distance of any point like P or Q from 0. … The idea of a continuous range, so familar to mathematicians in our days, is something quite exorbitant, an enormous extrapolation of what is really accessible to us. The idea that you should really indicate the exact values of any physical quantity – temperature, density, potential, field strenght, or whatever it might be – for all the points of a continuous range, say between 0 and 1, is a bold extrapolation. We never do anything else than determine the quantity approximaely for a very limited number of points and then „draw a smooth curve through them". This serves us well for many practical purposes, but from the epistemological point of view, from the point of the theory of knowledge, it is totally different from a supposed exact continual description.

(WeH), p. 1, "Preface", 1917: "At the center of my reflections stands the conceptual problem posed by the continuum - a problem which ought to bear the name of Pythagoras and which we currently attempt to solve by means of the arithmetical theory of irrational numbers".

(WeH1) p. 38: „In a different form than in the sequence of integers we encounter the infinite in the continuum, which is capable of infinite division. Cases of special importance are the continua of time and of space. Here we find the second open place in the above described construction of mathematical realm of numbers.",

(WeH1) p.40: „Since according to Eudoxus the ratio of any segment „a" to the unit segment determines a cut, the axiom of Dedekind guarantees the completeness of the geometrical elements: the system of points is incapable of extension, provided the axioms (including that of Eudoxus) are maintained (Hilbert)."

(WEH1) p. 41: „The essential character of the continuum is clearly described in this fragment due to Anaxagoras: „Among the small there is no smallest, but always something smaller. For what is cannot cease to be no matter how far it is being subdivided",

(WEH1) p. 41: „In contrast to this nature of the continuum, Leibniz conceives the idea of the monads, since – differently from Kant – he fells compelled to give the phenomena metaphysically a foundation in a world of absolute substances. Within the ideal or the continuum the whole precedes the parts."

(WeH1) p. 42: „Descartes struggles with the idea that the material corpuscles of a liquid in motion have to divide in infinitum", „or at least in indefinitum, and into so many parts that it is impossible to imagine one, however small, of which one would not know that it was actually subdivided into still smaller parts",

(WeH1) p. 42: „Hume is forced to admit that the „just as well as obvious" principle of comparing the measures of curves and surfaces by means of the number of component elements is, in fact, useless. B. Riemann in his lectures (RiB), states the alternative „that for a discrete manifold the principle of measurement is already contained in the concept of this manifold, but that for a continuous one it must come from elsewhere",

(WeH1) p. 42: „According to the first and most radical the continuum consists of countable discrete elements, atoms. With regards to matter, this path, initated by Democritus in antiquity, has been followed with brilliant success in modern physics. Plato, clearly conscious of the goal of „saving" the phenomenon by means oft he idea, was the first to design a consistent atomism with respect to space."

(WeH1) p. 43: „The second attempt is that of the infinitely small. This is discussed ingeniously and in detail on the first day of Glileo's „Discorsi". … The limiting process was victorious. For the limit is an indispensible concpet, whose importance is not affected by the acceptance or rejections of the infinitely small. …. AS amatter of fact, it is not impossible to build up a consistent „non-Archimedean" theory of quantities in which the axiom of Eudoxus (usually named after Archimedes) does not hold".

(WeH1) p. 45: „The third attempt to „save" the continuum in the Platonic sense may be seen in the modern set-theoretic foundations of analysis; the related mathematical conceppts are the onn-Archimedian, ordered field of hyper-real numbers and ist related non-standard analysis."

(WeH1), p. 86: "While topology has succeeded fairly well in mastering continuity, we do not yet understand the inner meaning of the restriction to differential manifolds. Perhaps one day physics will be able to discard it. At present it seems indispensable since the laws of transformation of most physical quantities are intimately connected with that of the differentials dx_i." ...

... "As the true lawfulness of nature, according to Leibniz's continuity principle, finds ist expression in laws of nearby action, connecting only the values of physical quantities at space-time points in the immediate vicinity of one another, so the basic relations of geometry should concern only infinitely closely adjacent points ('near-geometry' as opposed to far-geometry'). Only in the infinitely small may we expect to encounter the elementary and uniform laws, hence the world must be comprehended through its behavior in the infinitely small".

(*) The (mathematical) cardinality of that line is $2^{\aleph}$

Continuous Manifolds

(FlH) p. 49: „An n –dimensional manifold is a space which is not necessarily an Euclidean space nor is it a domain in a Euclidean space, but which, from the viewpoint of a short-sightes observer living in the space, looks just like such a domain of Euclidean space. … We have the technical problem of describing an n –manifold with sufficient precision so that we can define functions, tensors, and differential forms on such a space. … Each observer on the manifold has an immediate neighborhood (local coordinate neightborhood). Described by n coordinates. Each point in space must lie in at least one of these observed neighborhoods. Now if we consider simultaneously two observers, their immediate neighborhoods may overlap, and we must specify what happens in each such overlap.“

The history of manifolds is the attempt to build a mathematical structure to model the whole (the continuum) and the particular (the part) to put its combination then into relationship to describe motion, action etc. (Helmholtz, Riemann, Poincare, Lie and others). From (SchE) we recall the two conceptual design strategies for a "continuum":

Strategy I: Design of an "atomistic" theory of the continuum (which to H. Weyl's opinion contradicts to the essence of the continuum by itself)

Strategy II: develop a mathematical framework which symbolically explores the "relationship between the part and the whole" for the case of the continuum.

The later one leads to the concept of affine connexion, based on the concept of a manifold, which were developed during a time period of about 100 years, based on Riemann’s idea of a „*Geometry in Manifolds*“. This ia about a generalization of the classical concept of extended magnitudes/quantities for geometry and to "*construct*" the latter as only one specification from a more general concept. The terminology of "*multiple extended quantities*" was introduced by B. Riemann, synonymly to a "*continuous manifold*". It is conceptually based on two essential attributes: "*continuity*" and "*multiple extension*". From the extensional point of view such a concept would form a manifold and the individual modes of determination were to be considered, as the elements or the points of the manifold with either "discrete" or "continuous" transition from one to the other.

Scholz E., (SchE): 1.2. Riemann's n-dimensional manifolds

When Riemann presented his ideas on a geometry in manifolds the first time to a scientific audience in his famous Habilitationsvortrag (Riemann, 1854), he was completely aware that he was working in a border region between mathematics, physics, and philosophy, not only in the sense of the pragmatic reason that his audience was mixed, but by the very nature of his exposition. There was no linguistic or symbolical frame inside mathematics, which he could refer to, even only to formulate a general concept of manifold. So he openly drew on the resources of contemporary idealist, dialectical philosophy, in his case oriented at J.F. Herbart, to generalize the classical concept of extended magnitude/quantity for geometry and to "construct" the latter as only one specification from a more general concept. Basic to such a construction was, so Riemann explained to his audience, the presupposition of any "general concept" which allows in a logical sense precise individual determinations. From the extensional point of view such a concept would form a manifold and the individual modes of determination were to be considered, as Riemann explicitly stated, as the elements or the points of the manifold with either "discrete" or "continuous" transition from one to the other. Thus Riemann sketched the draft for a conceptual starting point for what later was to become general set theory (discrete manifolds) and topology (continuous manifolds). Such concepts would gain mathematical value only if a sufficiently rich structure of (real or complex valued) functions on the manifold is available. Then it should be possible to describe the specification of points by the values of n properly chosen functions in a locally unique way (local coordinate system). That a change of coordinates would lead to locally invertible differentiable real functions, was not made explicit by him, but was to be understood from the context by careful listeners or readers. The distinction between local simplicity of manifolds, because of the presupposition of local coordinate systems, and globally involved behaviour was indicated by Riemann, but not particularly emphasized during the talk, although in other pubhcations and manuscripts it was.

Of the utmost importance was Riemann's discussion of different conceptual levels - we would say structures - which can be considered on a given manifold. During his talk he exemplified these by the distinction between analysis situs (combinatorial topology of differential manifolds) and differential geometry. In his works on complex function theory he moreover pursued concrete investigations of complex and birational structure in the complex one-dimensional case (Riemann, 1851, 1857). And there are points in the latter publications, where Riemann indicated that it might be useful to work with even more "general concepts" of a continuous character, which would transcend the limits of the specific postulates for continuous manifolds introduced or at least presupposed in his Habilitations lecture. Thus in his dissertation Riemann (1851, p. 36) had already talked about infinite dimensional (real) function spaces and continuously varying conditions for functions in them, given by equations, which indicated nonlinear subsets in the dual of functions spaces. Moreover, Riemann had even already used the language of "continuous manifold" in this context without further specification what should be understood by that term. That was a drastic generalization of Gauss's finite dimensional linear submanifolds of R(n) and even far more general than the manifold concept as developed by Riemann in 1854.

(SchE2): Eine Mannigfaltigkeit war für Riemann demnach zunächst ein Konzept zur mathematischen Erfassung des Zustandsraumes eines (materiellen oder ideellen) Dinges. Wie später in seinem Habilitationsvortrag betonte Riemann auch schon hier, daß im allgemeinen keine Identifizierung der Mannigfaltigkeit mit reellen Intervallen möglich ist, und unterschied sprachlich zwischen „Größe" und „Zahl":

„Es wird hier aber durchaus nicht vorausgesetzt, dass es möglich sei, eine bestimmte Mannigfaltigkeit gleichsam als Maßstab für andere fortzutragen. Man hat sich daher vor der sonst geläufigen Vorstellung zu hüten, sich eine veränderliche Grösse durch eine veränderliche Zahl in eine feste Einheit ausgedrückt zu denken; es kann ferner eine Grösse einer Grösse nur insofern gleich sein, als sie mit ihr identisch ist und nur insofern kleiner, als sie ein Theil derselben ist".

An andere Stelle traf er eine Unterscheidung zwischen „endlichen" bzw. „unendlichen" (nicht nur eindimensionalen) Mannigfaltigkeiten: „Eine unendliche Mannigfaltigkeit würde er eine solche sein, von der man sich stets nur einen Theil vorstellen könnte, in dem, wie lange man sich auch neue Theile hinzudächte, doch nie die Gesamtheit erschöpft würde; wir beschränken uns aber in der Folge stets auf endliche Mannigfaltigkeiten.

Unter einem Teil der Mannigfaltigkeit M, „den man sich vorstellen kann", darf man wohl eine gleichdimensionale Untermannigfaltigkeit U verstehen, die durch eine offene und beschränkte Untermannigfaltigkeit von R, R^2, R^3 (oder allgemein R^n) darstellbar ist.

Einen breiten Raum widmete Riemann in den Notizen der Diskussion eindimensionaler Mannigfaltigkeiten. Dabei erklärte er Eindimensionalität dadurch, daß ein Übergang zwischen zwei Bestimmungsweisen/Zuständen des veränderlichen Dings (und damit zwischen zwei Punkten der Mannigfaltigkeit) „nur auf zweierlei Weise, vorwärts oder rückwärts" stattfinden könne. Erst nachdem das klargestellt war, führte er Parametrisierungen ein, die ihm eine Bezeichnung der Punkte der eindimensionalen Mannigfaltigkeit durch reelle Zahlen - keine Identifizierung - möglich machte. Er unterschied vier (topologische) Typen eindimensionaler Mannigfaltigkeiten:

- „beiderseits begrenzt" (vom Typ des abgeschlossenen Intervalls),
- „auf einer Seite begrenzt, auf einer unendlich" (halboffenes Intervall),
- „beidseitig unendlich" (offenes Intervall),
- „in sich zurücklaufend“.

General Relativity & The Ether of Lorentz

The fundamental principle of the SRT is the (Maxwell equations based) invariance principle building on the Lorentz transformation, which has a last mechanical quality, namely, its immobility, (EiA2).

Einstein's central message for regarding the relationship of ether and general relativity is

> *„The existence of the gravitational field is inseparably bound up with the existence of space. On the other hand a part of space may very well be imagined without an electromagnetic field"*.(*)

The alignment with a Hilbert space based extended SRT is enabled by the *complex* Lorentz group $L(C)$, which is also essential in the proof of the PCT theorem. The central differentiator to the Lorentz group is the fact, that $L(C)$ has the (only two) *connected* components $L_{+/-}(C)$, where $L_{+}(C)$ denotes the proper *complex* Lorentz group.

> (StR): *„For a general analysis of relativistic invariance it is reasonable that any relativistically invariant theory in which the states are spanned by the collision states of the elementary particles of the theory has, in a suitable basis, an essentially uniquely determined relativistic transformation law. This transformation law is identical to that of a theory of non-interacting elementary particles of the same masses and spins. Any relativistic theory of particles which does not have this transformation law will, in our opinion, require a novel physical interpretation. (as usual, in making this statement we are ignoring the special difficulties associated with zero mass particles.)"*

(EiA2) Einstein A., Ether and the theory of relativity, an address delivered on May 5th, 1920, in the University of Leyden

… Thus the endeavour towards a unified view of the nature of forces leads to the hypothesis of an ether. … in the first half of the nineteenth century the ether hypothesis was supported by the theory of the stationary luminiferous ether, which found fresh support in an experiment which is also of fundamental importance in the special theory of relativity, the experiment of Fizeau, from which one was obliged to infer that luminiferous ether does not take part in the movements of bodies. The phenomenon of aberration also favoured the theory of the quasi-rigid ether.

The development of the theory of electricity along the path opened up by Maxwell and Lorentz gave the development of our ideas concerning the ether quite a peculiar and unexpected turn. For Maxwell himself the ether indeed still had properties which were purely mechanical, although of a much more complicated kind than the mechanical properties of tangible solid bodies. But neither Maxwell nor his followers succeeded in elaborating a mechanical model for the ether which might furnish a satisfactory mechanical interpretation of Maxwell's laws of the electro-magnetic field. The laws were clear and simple, the mechanical interpretations clumsy and contradictory. Almost imperceptibly the theoretical physicists adapted themselves to a situation which, from the standpoint of their mechanical programme, was very depressing. They were particularly influenced by the electro-dynamical investigations of Heinrich Hertz. For whereas they previously had required of a conclusive theory that it should content itself with the fundamental concepts which belong exclusively to mechanics (e.g. densities, velocities, deformations, stresses) they gradually accustomed themselves to admitting electric and magnetic force as fundamental concepts side by side with those of mechanics, without requiring a mechanical interpretation for them. Thus the purely mechanical view of nature was gradually abandoned. But this change led to a fundamental dualism which in the long-run was insupportable. A way of escape was now sought in the reverse direction, by reducing the principles of mechanics to those of electricity, and this especially as confidence in the strict validity of the equations of Newton's mechanics was shaken by the experiments with β-rays and rapid Kathode rays.

Such was the state of things when H. A. Lorentz entered upon the scene. He brought theory into harmony with experience by means of a wonderful simplification of theoretical principles. He achieved this, the most important advance in the theory of electricity since Maxwell, by taking from ether its mechanical, and from matter its electromagnetic qualities. As in empty space, so too in the interior of material bodies, the ether, and not matter viewed atomistically, was exclusively the seat of electromagnetic fields. According to Lorentz the elementary particles of matter alone are capable of carrying out movements; their electromagnetic activity is entirely confined to the carrying of electric charges. Thus Lorentz succeeded in reducing all electromagnetic happenings to Maxwell's equations for free space.

As to the mechanical nature of the Lorentzian ether, it may be said of it, in a somewhat playful spirit, that immobility is the only mechanical property of which it has not been deprived by H. A. Lorentz. It may be added that the whole change in the conception of the ether which the special theory of relativity brought about, consisted in taking away from the ether its last mechanical quality, namely, its immobility. How this is to be understood will forthwith be expounded.

The space-time theory and the kinematics of the special theory of relativity were modelled on the Maxwell-Lorentz theory of the electromagnetic field. This theory therefore satisfies the conditions of the special theory of relativity, but when viewed from the latter it acquires a novel aspect. For if K be a system of co-ordinates relatively to which the Lorentzian ether is at rest, the Maxwell-Lorentz equations are valid primarily with reference to K. But by the special theory of relativity the same equations without any change of meaning also hold in relation to any new system of co-ordinates K′ which is moving in uniform translation relatively to K. Now comes the anxious question: -Why must I in the theory distinguish the K system above all K′ systems, which are physically equivalent to it in all respects, by assuming that the ether is at rest relatively to the K system? For the theoretician such an asymmetry in the theoretical structure, with no corresponding asymmetry in the system of experience, is intolerable. If we assume the ether to be at rest relatively to K, but in motion relatively to K′, the physical equivalence of K and K′ seems to me from the logical standpoint, not indeed downright incorrect, but nevertheless unacceptable.

More careful reflection teaches us, however, that the special theory of relativity does not compel us to deny ether. We may assume the existence of an ether; only we must give up ascribing a definite state of motion to it, i.e. we must by abstraction take from it the last mechanical characteristic which Lorentz had still left it. We shall see later that this point of view, the conceivability of which I shall at once endeavour to make more intelligible by a somewhat halting comparison, is justified by the results of the general theory of relativity.

In Minkowski's idiom this is expressed as follows: -Not every extended conformation in the four-dimensional world can be regarded as composed of world-threads. The special theory of relativity forbids us to assume the ether to consist of particles observable through time, but the hypothesis of ether in itself is not in conflict with the special theory of relativity. Only we must be on our guard against ascribing a state of motion to the ether.

What is fundamentally new in the ether of the general theory of relativity as opposed to the ether of Lorentz consists in this, that the state of the former (GRT) is at every place determined by connections with the matter and the state of the ether in neighbouring places, which are amenable to law in the form of differential equations; whereas the state of the Lorentzian ether in the absence of electromagnetic fields is conditioned by nothing outside itself, and is everywhere the same. The ether of the general theory of relativity is transmuted conceptually into the ether of Lorentz if we substitute constants for the functions of space which describe the former, disregarding the causes which condition its state. Thus we may also say, I think, that the ether of the general theory of relativity is the outcome of the Lorentzian ether, through relativation.

If we consider the gravitational field and the electromagnetic field from the stand-point of the ether hypothesis, we find a remarkable difference between the two. There can be no space nor any part of space without gravitational potentials; for these confer upon space its metrical qualities, without which it cannot be imagined at all. The existence of the gravitational field is inseparably bound up with the existence of space. On the other hand a part of space may very well be imagined without an electromagnetic field; thus in contrast with the gravitational field, the electromagnetic field seems to be only secondarily linked to the ether, the formal nature of the electromagnetic field being as yet in no way determined by that of gravitational ether. From the present state of theory it looks as if the electromagnetic field, as opposed to the gravitational field, rests upon an entirely new formal motif, as though nature might just as well have endowed the gravitational ether with fields of quite another type, for example, with fields of a scalar potential, instead of fields of the electromagnetic type.

(*) (StR): „*The corresponding Lorentz group L has four disconnected components, where each of which is connected in the sense that any one point can be connected to any other, but no Lorentz transformation in one component can be connected to another in another component. This results to three subgroups of L, which are the orthochronous Lorentz group, the proper Lorentz group, and the orthochorous Lorentz group. Associated with the restricted Lorentz group is the group of 2x2 complex matrices of determinant one ($SL(2,C)$).*"

Space & Time & Matter

Some quotes

(ScE1) p.141: „I say, the most appreciable help science has offered us (in this) is, in my view, the gradual idealization of time. In thinking of this the names of three men obtrude themselves upon us, though many others, including non-scientists, have hit on the same groove, such as St Augustine of Hippo and Boethius; the three are Plato, Kant and Einstein. … In Plato's case it came from mathematics and geometry. … In my opinion it was this, that he was the first to evisage the idea of timeless existence and to enphasize it – against reason – as a reality, more real than our actual experience; this he said, is but a shadow of the former, from which all experienced reality is borrwed. I am speaking of the theory of forms (or ideas).

(ScE1): p. 144: „Let us now turn to Kant. It has become a commonplace that he thaught the ideality of space and time and that this was a fundamental, if not the most fundamental part of teaching. Like the most of it, it can be neither verified nor falsified, but it does not lose interest on this account (rather it gains; if it could be proved or disproved it would be trivial). The meaning is that, to be spread out in space and to happen in a well-defined temporal order of „before and after" is not a quality of the world that we perceive, but pertains to the perceiving mind which, in its present situation anyhow, cannot help registering anything that is offered to it according to these two card-indexes, space and time. It does not mean that mind comprehends these order-schemes irrespective of, and before, any experience, but that it cannot help developing them and applying them to experience when it comes along, and particularly that this fact does not prove or suggest space and time to be an order-scheme inherent in that „thing-in-itself" which, as some believe, causes our experience."

(ScE1) p. 146: „I now come to speak about Einstein in the same context. … The new view has its strongest impact on the previous notion of time. Time is the notion of „before and after". The new attitude springs from the following two roots: (i) The notion „before and after" resides on the „cause and effect" relation. …(ii) The second root is the experiemental and observational evidence that effects do not spread with arbitrarily high velocity. There is an upper limit, which incidentally is the velocity of light in empty space."

(ScE3): The two underlying paradigms of the geometric structure of the space-time model of the special relativity theory are

i) the equivalence of all four-dimensional systems of coordinates obtained from any of them by arbitrary (point-) transformation,

ii) the continuum has a metrical connexion impressed on it, that is at every point a certain quadratic form of coordinate-differentials, $g_{ik}dx_idx_k$ called, the „square of the interval" between the two points in question, has a fundamental meaning, invariant in the aforesaid transformations.

These two principles are of very different standing. The first, the principle of general invariance, incarnates the idea of General Relativity. … One has occcasionally tried to generalize it, and it is difficult to say whether quantum physics might not at some time seriously dictate its generalization. … An important group of attempts to generalize the 1915 theory is based on this more general tpye of connexion. We shall therefore investigate the geometry of our continuum in three steps or stages, viz.

i) when only general invariance is imposed;

ii) when in addition an affine connexion is imposed;

iii) when this is specialized to carry a metric.

(SuL) 1.6: Einstein took Maxwell's equations to be the law of physics. He knew that they give rise to wavelike solutions. At age sixteen, he puzzled over what would happen if you moved along with the light ray. The „obvious" answer is that you would see a static electric and magnetic field with a wavelike structure that does not move. Somehow, he knew that was wrong – that it was not a solution to Maxwell's equations. Maxwell's equations say that light moves at the speed of light. …. In modern language we would explain Einstein's reasoning a little different. We would say that Maxwell's equations have a symmetry of some kind – some set of coordinate transformations under which the equations have the same form in every reference frame. …

(UnA2) Chapter 4, The Basis Story, What Einstein told us about gravity and space-time

According to the „standard" or „concordance" model of cosmology, the universe is made up of 4 percent usual matter, while the rest of invisible substances such as dark matter and dark energy. The vast majority of astronomers consider this model to be an essentially correct description of the cosmos. However one gives the concordance model too much credit by viewing it as a unique consequence of Einstein's celebrated theory of general relativity found in 1915. All of its equations relevant to cosmology can be derived from Newton's laws as well. For the small accelerations that govern cosmological dynamics, Einstein's theory perfectly merges into Newton's law for weak gravitational fields. The same holds for Newton's laws of motion and the theory of special relativity that Einstein had come up with already in 1905. Relativity builds upon a surprisingly simple insight – the speed of light is always the same, even if one views it from a moving system like a car or a rocket. … There aren't the slightest doubts about the theory of special relativity. All of its predictions, such as high speeds leading to a time dilation, or mass increase conveyed in the famous equation $\mathrm{E} = \mathrm{m} \cdot c^2$, are in excellent agreement with the observations.

Nature's timetable: fixed by special and general relativity

No wonder that principles such as that of least time, developed and put to good use by the mathematicians Euler and Lagrange, are considered benchmarks for finding fundamental laws of the nature. And, of course, there is a crucial role for energy when we are talking about time and motion. It turns out that the sum of kinetic, potential, and other forms of energy are conserved at all times and during any event. It does no matter whether it is the swinging of pendulums, the oscillation of springs, or the falling of a stone. The concept of energy is a remarkable powerful way to help us analyze the processes regardless of the details of motion and time of the moment. However, for this reason, energy conservation is basically a rule grown out of the desire to have time-independent laws of nature. But are they really time independent, if we consider cosmological periods?

Barbour's central idea is that time is defined through the various periodicities we observe in Nature. It is a profound generalization of Mach's principle. …. The cosmologist John Barrow has noted,

> *„The question if there is a unique absolute standard of time which globally is defined by the inner geometry of the universe, is a big unresolved problem of cosmology".*

General Relativity & Quantum Mechanics

Some quotes from

(AnE): Anderson E., The Problem of Time, Quantum Mechanics versus General Relativity

(KaM) Kaku M., Introduction to Superstrings and M-Theory, Springer Verlag, 1988, 1999

> (AnE): „*In approaching Quantum Gravity, many conceptual issues turn out to be related to notions of time. This occurs because notions of time are substantially different across the Newtonian Physics, Special Relativity (SR), Quantum Mechanics (QM), Quantum Field Theory (QFT) and General Relativity (GR). A first example in which this occurs is QM versus GR. …. The problem of time is, in greater generality, a consequence of the missmatch between Background Dependent and Background Independent Paradigms of Physics. Newtonian Physics, SR, QM, and QFT are all Background Dependent, whereas GR is Background Independent and many approaches to Quantum Gravity expect this to be Background Independent as well*."
>
> *Smolin L.: „Einstein, both a realist and a physicist, believed that it was necessary to go beyond quantum mechanics to discover what was missing from a true theory of atoms. This was Einstein's unfinished mission", (SmL)*
>
> *Smolin L.: „Bohm discussed interpretational issues he kept close to the Copenhagen orthdoxy. One section of his (Bohm's) book was titled „Proof that quantum theory is inconsistent with hidden variables", (SmL) book cover*

In order to motivate the superstring theory in (KaM) it is pointed out, that because general relativity and quantum mechanics can be derived from a small set of postulates, and the theories are incompatible one or more of these postulates must be wrong:

> (KaM) p. 8: „Gravity research was totally uncoupled from research in other (weak, strong elementary particles) interactions. Classical relativists continued to find more and more classical solutions in isolation from particle research. Attempts to cannonically quantize the theory were frustrated by the presence of the tremendous redundancy of the theory. There was also the discouraging realization that even if the theory could be successfully quanized, it would still be nonrenormalizable.
>
> (KaM) p. 9: General relativity is (also) plagued with similar difficulties when pushed to ist limits:
>
> (1) … Einstein's equations necessarily exhitbit pointlike singularities, where we expect the laws of general relativity to collapse. Quantum corrections must dominate over the classical theory in this domain
>
> (2) The action is not bounded from below, it is linear in the curvature tensor. Thus, it may not be stable quantum mechanically
>
> (3) General relativity is not renormalizable. Computer calculations, for example, have now conclusively shown that there is a nonzero counterterm in Einstein's theory at the two-loop level.
>
> (KaM) p. 12: Physicists have concluded that perhaps one or more of our cherished assumptions about our Universe must be abandoned. Because general relativity and quantum mechanics can be derived from a small set of postulates, one or more of these postulates must be wrong. The key must be to drop one of our commonsense assumptions about Nature on which we have constructed general relativity and quantum mechanics. Over the years, several proposals have been made to drop some of our commonsense notions about the Universe (for more detail we refer to the appendix):
>
> (1) Continuity
> (2) Causality
> (3) Unitary
> (4) Locality
> (5) Point Particle.
>
> The super string theory, because it abandons only the assumption that the fundamental constituents of matter must be point particles, does the least amount of damage to cherished physical principles and continues the tradition of increasing the complexity and sophistication of the gauge group. Superstring theory does not violate any of the laws of quantum mechanics, yet manages to eliminate most, if not all, of divergences of the Feynman diagrams."

Quantum Mechanics & Quantum Field Theory

A physical system is built by a series of physical components, which can be different kinds of particles or fields.

For a mechanical system (particles with mass and interacting forces) the equations of motion are Ordinary Differential Equations (ODE), while for a dynamical system (waves, field forces, energy densities, energy flows) the equations of motion are Partial Differential Equations.

We note that ODE and only parabolic / hyperbolic PDE are accompanied with the notion of *time*. In case of ODE and parabolic PDE this notion comes along with the concept of oa „*time arrow*“, while a hyperbolic PDE comes along with the notion of „*time reversibilty*“.

In quantum theory the mechanical system is the quantum mechanics; the dynamical system is the quantum field theory. The still fundamental challenge to compbine both systems has been stated by E. Fermi (FeE):

> „*Dirac's theory of radiation is based on a very simple idea; instead of considering an atom and the radiation field with which it interacts as two distinct systems, he treats them as a single system whose energy is the sum of three terms: one representing the energy of the atom, a second representating the electromagnetic energy of the radiation field, and a small term representing the coupling energy of the atom and the radiation field*.“

The model replaces Dirac‘s model of the „*charge of a point particle*“. Mathematically speaking, it replaces the Dirac „function“, which is an element of the Hilbert space $H_{-n/2-\varepsilon}$ (n denotes the space dimension, and $\varepsilon > 0$), by „*quantum elements*“ of the smaller Hilbert space $H_{-1/2}$ (independently from the space dimension), where the complementary space $H_1^{\perp}$ of the split $H_{1/2} = H_1 \otimes H_1^{\perp}$ is governed by wavelets.

(*) (EiA) p. 52 (translation by the author): „„However, the laws governing the currents and charges (in the Maxwell equations), are unknown to us. We know, that electricity exists within elementary particles (electrons, positive kernels), but we don't understand it from a theoretical perspective. We do not know the energetical factors, which determine the electricity in particles with given size and charge; and all attempts failed to complete the theory in this directions. Therefore, if at all we can built on the Maxwell equations, we know the energy tensor of electromagnetic fields only outside of the particles“.

(**) (UnA): Einstein quote: … „Nothing forces us to assume that … clocks have to be seen as running at the same speed“

(ToA) p. 21, H. Berson, "Damit erhält man zwei Ausdehnungen: einmal eine Ausdehnung, in der sich die endlichen Dinge bewegen, während die andere Ausdehnung als Unendliches, Homogenes betrachtet wird. .... Die Bewegung ist wie die wirkliche Zeit nicht teilbar und auch nicht auf den Raum reduzierbar. Sie ist im wahrsten Sinne des Wortes nur mit der Dauer möglich. In diesem Sinne hat die Bewegung mit der teilbaren, messbaren Zeit wenig zu tun."

(ReW), "Für Heidegger ist es die Zeit, die das Sein und damit den Sinn von Sein konstituiert. Wesentlich für das Verständnis von Sein ist die Einsicht, dass das Sein unzertrennbar mit dem Nichts verknüpft, ja davon durchzogen ist. Unser Dasein ist wesentlich durch die Möglichkeit und die Realität des Nicht-seins bestimmt. Der Tod ist die jede Sekunde präsente Möglichkeit des Nicht-seins. Realität ist das Nicht-sein selbstverständlich in dem Sinn, dass wir wissen, dass es uns lange Zeit vor unserer Geburt nicht gegeben hat und dass es eine lange Zeit geben wird, in der wir nicht mehr da sein werden.

Bound Electrons

Some quotes about „bound electrons“ from W. Heisenberg in the context of „Critique oft he corpuscular theory, (HeW):

(HeW) p. 13: „The concepts of velocity, energy, etc., have been developed from simple experiments with common objects, in which the mechanical behavior of macroscopic bodies can be described by the use of such words. These same concepts have then been carried over to the electron, since in certain fundamental experiments electrons show a mechanical behavior like that of the objects of common experience. Since it is known, however, that this similarity exists only in a certain limited region of phenomena, the applicability of corpuscular theory must be limited in a corresponding way. According to Bohr, this restriction may be deduced from the principle that the processes of atomic physics can be visualized equally well in terms of waves or particles. Thus the statement that the position of an electron is known to within a certain accuracy Δx at the time t can be visualized by the picture of a wave packet in the proper position with approximate extension Δx. By „wave packet“ is meant a wavelike disturbance whose amplitude is appreciably different from zero only in a bounded region. This region is, in general, in motion, and also changes its size and shape, i.e., the disturbance spreads. The velocity of the electron corresponds to that of the wave packet, but this latter cannot be exactly defined, because of the diffusion which takes place. This indeterminateness is to be considered as an essential characteristic of the electron, and not as evidence of the inapplicability of the wave picture.“

(HeW) p. 30: „If it is required to deduce the uncertainty relation for the position q, and the momentum p, of bound electrons, two problems must be clearly distinguished. The first assumes that the energy of the system, i.e., its stationary state, is known, and then inquires what accuracy of knowledge of p and q is implied in, or is incompatible with, this knowledge of energy. The second, distinct problem disregards the possibility of determining the energy of the system and merely inquires what the greatest accuracy is with which of p and q may simultaneously be known. In this second case, the eperiments necessary for the measurement of p and q may produce transitions from one stationary state to another; in the first case, the methods of measurement must be so chosen that transitions are not induced.“

(HeW) p. 36: „critique of the corpuscular theory“: „The motion and spreading of probability packets has been studied by various authors, … A simple consideration of Ehrenfest's may be mentioned, … considering the motion of a single electron moving in a field of force whose potential is $V(q)$. … If there were no spreading at all, it would be possible to make a Fourier analysis of the probability density into which only integral multiples of the fundamental frequency of the orbit enter.

(HeW) p. 38: „A remark concerning the rate of sreading of the wave packet may not be out of place at this point. If the classical motion of the system is periodic, it may happen that the size of the wave packet at first undergoes only periodic changes. The number of revolutions which the packet may perform before it spreads completely over the whole region of the atom can be calculated qualitatively as follows: if there is no spreading at all, it would be possible to make a Fourier analysis of the probability density into which only integral multiples of the fundamental frequency of the orbit enter. As a matter of fact, however, the „overtones“ of quantum theory are not exactly integral multiples of this fundamental frequency. The time in which the phase of the quantum theoretical overtones is completely shifted from that of the classical overtones will be qualitatively the same as the time required fo the spreading of the wave packet. Let J be the action variable of classical theory, then this time will be

$$t \sim \frac{1}{h\frac{\partial v}{\partial J}} ,$$

and the number of revolutions performed in this time is

$$N \sim \frac{v}{h\frac{\partial v}{\partial J}} .$$

In the special case of the harmonic oscillator, N becomes infinite – the wave packet remains small for all time. In general, however, N will be of the order of magnitude of the quantum number n.

In relation to these considerations, one other idealized experiment (due to Einstein) may be considered. We imagine a photon which is represented by a wave packet built up out of Maxwell waves. It will thus have a certain spatial extension and also a certain range of frequency. By reflection at a semi-transparent mirrow, it is possible to decompose it into two parts, a refleted and a transmitted packet. There is then a definite probability for finding the photon either in one part or in the other part of the divided wave packet. After a sufficient time the two parts will be separated by any distance desired; now if an experiment yields the result that the photon is, say, in the reflected part of a packet, then the probability of finding the photon in the other part of the packet immediately becomes zero. The experiment at the position of the reflected packet thus exerts a kind of action (reduction of the wave packet) at the distance point occupied by transmitted packet, and one sees that this action is propagated with velocity greater than that of light. However, it is also obvious that this kind of action can never be utilized for the transmission of signals so that it is not in conflict with the postulates of the theory of relativity.“

The D'Alembert Paradoxon

The d'Alembert "paradox" is not about a real paradox but it is about the failure of the Euler equation (the model of an ideal incompressible fluid) as a model for fluid-solid interaction. The difficulty with ideal fluids and the source of the d'Alembert paradox is that in incompressible fluids there are no frictional forces. Two neighboring portions of an ideal fluid can move at different velocities without rubbing on each other, provided they are separated by streamline.

The problem is that such a phenomenon can never occur in a real fluid, and the question is how frictional forces can be introduced into a model of a fluid.

The Navier-Stokes Equations describes a flow of incompressible, viscous fluid. They are derived from the (Cauchy) stress tensor (resp. the shear viscosity tensor) leading to liquid pressure forces in the form $\frac{\partial p}{\partial x_i} = -\frac{\partial T_{ji}}{\partial x_j} + \mu \Delta v_i$.

The Navier–Stokes existence and smoothness problem for the three-dimensional NSE, given some initial conditions, is to prove that smooth solutions always exist, or that if they do exist, they have bounded energy per unit mass. Thereby, the Serrin gap occurs in case of space dimension $n = 3$ as a consequence of the Sobolev embedding theorem with respect to the energy Hilbert space H_1 with the Dirichlet integral as its inner product.

Mathematically speaking, the problem is to establish a well-posed 3D non-linear, non-stationary initial-boundary problem, e.g. with solutions w/o blow-up phenomenon and bounded energy inequality for $t \in [0, T]$ and $\vec{x} \in \Omega$.

In airfoil lift theory there is the concept of a "*vortex line*" generating a corresponding "*circuit/ vortex (uplift) force*". The theory delivers results, which matches quite good with experimental results. However it remains the difficulty to explain the origin of the circulation, based on the assumption that the "fluid" (the air) is considered as homogeneous medium, whereby in its equilibrium state no shear stresses can be transmitted. Considering air as an incompressible, frictonless medium then a body, which moves through this ideal "liquid" does not experience any resulting forces (Helmholtz, Kirchhoff). This miss match between theoretical assumptions and experimental results manifests in the question of the root cause of the circulation.

The variational formulation $a(u, v) = l(v)$ of the considered initial (integral) boundary problems in an extended $H_{1/2}$ energy norm framework is accompanied with „frictional forces". The extended framework fits perfectly into the theory of hypersingular integral operators in the context of aerodynamics and its underlying Prandtl operator.

The corresponding variational representation of the 3D-non-linear, non-stationary NSE initial boundary problem shows bounded „generalized" energy norm estimates in the form

$$\frac{1}{2}\frac{d}{dt}\|u\|_{-1/2}^2 + \|u\|_{1/2}^2 \leq \left|(Bu, u)_{-1/2}\right| \leq \|u\|_{-1/2}\|Bu\|_{-1/2} \cong \|u\|_{-1/2}\left\|A^{-1/4}Bu\right\|_0.$$

Putting $y(t) := \|u\|_{-1/2}^2$ one gets $y'(t) \leq c \cdot \|u\|_1^2 \cdot y^{1/2}(t)$, resulting into the a priori estimate

$$\|u(t)\|_{-1/2} \leq \|u(0)\|_{-1/2} + \int_0^t \|u\|_1^2(s)ds \leq c\left\{\|u_0\|_{-1/2} + \|u_0\|_0^2\right\},$$

which ensures global boundedness by the a priori energy estimate provided that $u_0 \in H_0$.

The crucial lemma to derive this estimate is

> *(GiY)* lemma 3.2.*:* For $0 \leq \delta < 1/2 + n \cdot (1 - 1/p)/2$ it holds $\left|A^{-\delta}P(u, grad)v\right|_p \leq M \cdot \left|A^{\theta}u\right|_p \cdot \left|A^{\rho}u\right|_p$ with a constant $M := M(\delta, \theta, \rho, p)$ if $\delta + \theta + \rho \geq n/2p + 1/2$, $\theta, \rho > 0$, $\theta + \rho > 1/2$. Putting $p := 2$, $\delta := 1/4$,$\theta := \rho := 1/2$ fulfilling $\theta + \rho \geq \frac{1}{4}(n + 1) = 1$ it follows
>
> $$\left\|A^{-\delta}P(u, grad)u\right\| \leq c\left\|A^{\theta}u\right\| \cdot \|A^{\rho}u\| = c\|u\|_{2\theta} \cdot \|u\|_{2\rho} = c\|u\|_1^2$$
>
> resp.
>
> $$\frac{1}{2}\frac{d}{dt}\|u\|_{-1/2}^2 + \|u\|_{1/2}^2 \leq \left|(Bu, u)_{-1/2}\right| \leq c \cdot \|u\|_{-1/2}\|u\|_1^2.$$

Mind & Matter

This section is about the notions of consciousness & inconsciousness and differentials & particles.

> (ScE): „Probably for historical reasons of language and education, it comes naturally to the simple man of today to think of a dualistic relationship between mind and matter as an extremly obvious idea. … But a more careful consideration should make us less ready to admit this interaction of event in two different spheres – if they really are different spheres; for the first (the causal determination of matter by mind) would necessarily have to disrupt the autonomy of material events, while the second (causal influence on mind of bodies or their equivalent, for example light) is absolutely unintelligible to us; in short, we simply cannot see how material events can be transformed into sensation or thought, however many text books, in defiance of Du Bois Reymond, go on talking nonsense on the subject.“
>
> (ScE) p. 42: „Thus Schopenhauer's line of demarcation may be regarded as highly suitable, when he says that in inorganic being „the essential and permanent element, the basis of identity and integrity, is the material, the matter, the inessential and mutable element being the form. For its life, that is, its existence as an organic being, consits precisely in a constant change of matter while the form persits“.
>
> (ScE) p. 45: „We will now return to the question asked at the beginning of the last chapter – „Which material events are directly associated with consciousness?“ – but starting this time from the somewhat surer ground of inner experience. We first try to show, on general grounds, that the idea that association is a unique prerogative of the functions of the brain is not very probable; and we then had to admit that attempts at extending the association to other events unfortunately lose themselves in vague, fantastic speculations. We now propose to make an observation of an opposite kind, but equallly capable of shattering this idea. It runs as follows:
>
> Not all brain-processes are accompanied by consciousness. There are nerve-processes which, while exactly resembling the „conscious“ processes of the brain both in their whole centripetal-centrifugal pattern and in their biological significance as reaction-regulators, nevertheless are not associated with consciousness. …. It seems to me that the key to this lies in the well-known fact that any particular series of phenomena in which we consciously or even actively participate, if it is repeated over and over again in exactly the same way, gradually sinks out of the sphere of consciousness; and it is only, so to speak, gragged up into it again if, on a fresh repetition, the event initiating the process, or the conditions affecting its continuance, are slightly different, in which case the reactions happen slightly differently too. But even then it is not the process as a whole, but only (primarily at least) the modifications or differentials, by which the new series is distinguished from the earlier ones, which enter into consciousness…
>
> In order to appreciate the significance of this gradual fading.out of consciousness in our mental life, we need to realise the enormous part played in it by training through repetition, mneme. Biologically speaking, a single experience is entirely insignificant; only efficient functioning in relation to frequently repeated situations is biologically valuable. …. Now let think of an organism confronted by a new biological situation. It react in a certain way and maintains itself by doing so, or at any rate is not destroyed. If the stimulus is repeated the same sequence is reproduced, which we will suppose is in the first place of such a nature as to enter into consciousness. Repwtion will then introduce something new into consciousness, the element of „having had this already“. But with frequent repetition the performance becomes better and better, as our inner experience shows; it gets less and less „interesting“, the reaction becomes more and more reliable, but also proportionately less and less conscious. Now suppose a change (differential) occurs in the external situation. This, or rather the difference in the reaction which it causes, gets through to the consciousness. But again, only so long as it is new. Gradually it too works in and sinks beneath the surface. Nor need the differential consist in a single and thereafter persistent change in the situation and its result; it may and often will be the case that the situation is modified now in one way and now in another; causing the reaction to be modified in a corresponding way. This kind of bifurcation gets worked in too: after a sufficient number of repetitions, the process of deciding in a particular case which situation is happening and how to react to it is performed quite unsciously. Then a second level of differential can be superimposed on the first, then a third, and so on. And so ad infinitum, the only differentials which get through to the consciousness being the most recent ones, the ones about which the living tissue is still „in training“.
>
> (ScE) p. 50: „It is the individual peculiarities in any one ontogenesis which become conscious. …
> Briefly summarising, we can express the proposed law thus: consciousness is bound up with learning in organics substance; organic competence is unconscious. Still more briefly, and put in a form which is admittedly rather obscure and open to missunderstanding: Becoming is conscious, being unsconsious.

Cosmic & Morally Religion

This section provides the section „Religion und Wissenschaft“ from Einstein’s view of the world“, (EiA1).

This german text seems not having found its way into other language translations, by chance or by purpose. In any case, facing his concept of „cosmic religion“ one of his most cited quote „“God“ (!) does not play dice“, (in the context of Bohr’s wave – particle dualism) just sound ironically to the concerned physical colleagues, very much in line with Schrödinger’s „cat“ message to the same audiance. For more concrete messages to this audiance we refer to (UnA2).

Religion und Wissenschaft

Alles, was von Menschen getan und erdacht wird, gilt der Befriedigung gefühlter Bedürfnisse sowie der Stillung von Schmerzen. Die muß man sich immer vor Augen halten, wenn man geistige Bewegungen und Entwicklung verstehen will. Denn Fühlen und Sehnen sind der Motor allen menschlichen Strebens und Erzeugens, mag sich uns letzteres auch noch so erhaben darstellen. Welches sind nun die Gefühle und Bedürfnisse, welche die Menschen zu religiösem Denken und zum Glauben im weitesten Sinne gebracht haben? Wenn wir hierüber nachdenken, so sehen wir bald, daß an der Wiege des religiösen Denkens und Erlebens die verschiedenen Gefühle stehen. Beim Primitiven ist es in erster Linie die Furcht, die religiöse Vorstellungen hervorruft. Furcht vor Hunger, wilden Tieren, Krankheit, Tod. Da auf dieser Stufe des Daseins die Einsicht für kausale Zusammenhänge gering zu sein pflegt, spiegelt uns der menschliche Geist selbst mehr oder minder analoge Wesen vor, von deren Wollen und Wirken die gefürchteten Erlebnisse abhängen. Man denkt nun, die Gesinnung jener Wesen sich günstig zu stimmen, indem man Handlungen begeht und Opfer bringt, welche nach dem von Geschlecht zu Geschlecht überlieferten Glauben jene Wesen besänftigen bzw. dem Menschen geneigt machen. Ich spreche in diesem Sinne von Furcht-Religion. Diese wird nicht erzeugt, aber doch wesentlich stabilisiert durch die Bildung einer besonderen Priesterkaste, welche sich als Mittlerin zwischen den gefürchteten Wesen und dem Volke ausgibt und hierauf eine Vormachtstellung gründet. Oft verbindet der auf anderen Faktoren sich stützende Führer oder Herrscher bzw. eine privilegierte Klasse mit ihrer weltlichen Herrschaft zu deren Sicherung die priesterlichen Funktionen, oder es besteht eine Interessengemeinschaft zwischen der politisch herrschenden Kaste und der Priesterkaste.

Eine zweite Quelle religiösen Gestaltens sind die sozialen Gefühle. Vater und Mutter, Führer größerer menschlicher Gemeinschaften sind sterblich und fehlbar. Die Sehnsucht nach Führung, Liebe und Stütze gibt den Anstoß zur Bildung des sozialen bzw. des moralischen Gottesbegriffes. Es ist der Gott der Vorsehung, der beschützt, bestimmt, belohnt und bestraft. Es ist der Gott, der je nach dem Horizont des Menschen das Leben des Stammes, der Menschheit, ja das Leben überhaupt liebt und fördert, der Tröster in Unglück und ungestillter Sehnsucht, der die Seelen der Verstorbenen bewahrt. Dies ist der soziale oder moralische Gottesbegriff.

In der heiligen Schrift des jüdischen Volkes läßt sich die Entwicklung von der Furcht-Religion zur moralischen Religion schön beobachten. Ihre Fortsetzung hat sie im Neuen Testament gefunden. Die Religionen aller Kulturvölker, insbesondere auch Völker des Orients, sind in der Hauptsache moralische Religionen. Die Entwicklung von der Furcht-Religion zur moralischen Religion bildet einen wichtigen Fortschritt im Leben der Völker. Man muß sich vor dem Vorurteil hüten, als seien die Religionen der Primitiven reine Furcht-Religionen, diejenigen der kultivierten Völker reine Moral-Religionen. Alle sind vielmehr Mischtypen, so jedoch, daß auf den höheren Stufen sozialen Lebens die Moral-Religion vorherrscht.

All diesen Typen gemeinsam ist der anthropomorphe Charakter der Gottesidee. Über diese Stufe religiösen Erlebens pflegen sich nur besonders reiche Individuen und besonders edle Gemeinschaften wesentlich zu erheben. Bei allen aber gibt es noch eine dritte Stufe religiösen Erlebens, wenn auch selten in reiner Ausprägung; ich will sie als kosmische Religiosität bezeichnen. Diese läßt sich demjenigen, der nichts davon besitzt, nur schwer deutlich machen, zumal ihr kein menschenartiger Gottesbegriff entspricht.

Das Individuum fühlt die Nichtigkeit menschlicher Wünsche und Ziele und die Erhabenheit und wunderbare Ordnung, welche sich in Natur sowie in der Welt offenbart. Es empfindet das individuelle Dasein als eine Art Gefängnis und will die Gesamtheit des Seienden als ein Einheitliches und Sinnvolles erleben. Ansätze zur kosmischen Religiosität finden sich bereits auf früher Entwicklungsstufe, z.B. in manchen Psalmen Davids sowie bei einigen Propheten. Viel stärker ist die Komponente kosmischer Religiosität im Buddhismus, was uns besonders Schopenhauers wunderbare Schriften gelehrt haben. – Die religiösen Genies aller Zeiten waren durch diese kosmische Religiosität ausgezeichnet, die keine Dogmen und keinen Gott kennt, der nach dem Bild des Menschen gedacht wäre. Es kann daher auch keine Kirche geben, deren hauptsächlicher Lehrinhalt sich auf die kosmische Religiosität gründet. So kommt es, daß wir gerade unten den Häretikern aller Zeiten Menschen finden, die von dieser höchsten Religiosität erfüllt waren und ihren Zeitgenossen oft als Atheisten erschienen, manchmal auch als Heilige. Von diesem Gesichtspunkt aus betrachte, stehen Männer wie Demokrit, Franziskus von Assisi und Spinoza einander nahe. Wie kann kosmische Religiosität von Mensch zu Mensch mitgeteilt werden, wenn sie doch zu keinem geformten Gottesbegriff und zu keiner Theologie führen kann? Es scheint mir, daß es die wichtigste Funktion der Kunst und der Wissenschaft ist, dies Gefühl unter den Empfänglichen zu erwecken und lebendig zu erhalten.

So kommen wir zu einer Auffassung von der Beziehung der Wissenschaft zur Religion, die recht verschieden ist von der üblichen. Man ist nämlich nach der historischen Betrachtung geneigt, Wissenschaft und Religion als unversöhnliche Antagonisten zu halten, und zwar aus leichtverständlichem Grund. Wer von kausalen Gesetzmäßigkeiten allen Geschehens durchdrungen ist, für den ist die Idee eines Wesens, welche in den Gang des Weltgeschehens eingreift, ganz unmöglich – vorausgesetzt allerdings, daß er es mit der Hypothese der Kausalität wirklich ernst nimmt. Die Furcht-Religion hat bei ihm keinen Platz, aber ebenso wenig die soziale bzw. moralische Religion. Ein Gott, der belohnt und bestraft, ist für ihn schon darum undenkbar, weil der Mensch nach äußerer und innerlicher Notwendigkeit handelt, vom Standpunkt Gottes also nicht verantwortlich wäre, so wenig wie ein lebloser Gegenstand für die von ihm ausgeführten Bewegungen. Man hat deshalb schon der Wissenschaft vorgeworfen, daß sie die Moral untergrabe, jedoch gewiß mit Unrecht. Das ethische Verhalten des Menschen ist wirksam auf Mitgefühl, Erziehung und soziale Bindung zu gründen und bedarf keiner religiösen Grundlage. Es stünde traurig um die Menschen, wenn sie durch Furcht vor Strafe und Hoffnung auf Belohnung nach dem Tode gebändigt werden müßten.
Es ist also verständlich, daß die Kirchen die Wissenschaft von jeher bekämpft und ihre Anhänger verfolgt haben. Andererseits aber behaupte ich, daß die kosmische Religiosität die stärkste und edelste Triebfeder wissenschaftlicher Forschung ist. Nur wer die ungeheuren Anstrengungen und vor allem die Hingabe ermessen kann, ohne welche bahnbrechende wissenschaftliche Gedankenschöpfungen nicht zustande kommen können, vermag die Stärke des Gefühls zu ermessen, aus dem allen solche dem unmittelbar praktischen Leben abgewandte Arbeit erwachsen kann. Welch ein tiefer Glaube an die Vernunft des Weltenbaues und welche Sehnsucht nach dem Begreifen wenn auch nur eines geringen Abglanzes der in dieser Welt geoffenbarten Vernunft mußte in Kepler und Newton lebendig sein, daß sie den Mechanismus der Himmelsmechanik in der einsamen Arbeit vieler Jahre entwirren konnten! Wer die wissenschaftliche Forschung in der Hautsache nur aus ihren praktischen Auswirkungen kennt, kommt leicht zu einer ganz anderen Auffassung vom Geisteszustand der Männer, welche – umgeben von skeptischen Zeitgenossen – Gleichgesinnten den Weg gewiesen haben, die über die Länder der Erde und über die Jahrhunderte verstreut waren. Nur wer sein Leben ähnlichen Zielen hingegeben hat, besitzt eine lebendige Vorstellung davon, was diese Menschen beseelt und ihnen die Kraft gegeben hat, trotz unzähliger Mißerfolge dem Ziel treu zu bleiben. Es ist die kosmische Religiosität, die solche Kräfte spendet. Ein Zeitgenosse hat nicht zu Unrecht gesagt, daß die ernsthalten Forscher in unserer im allgemeinen materialistisch eingestellten Zeit die einzigen tief religiösen Menschen seien.

References

(AnE) Anderson E., The Problem of Time, Springer, Cambridge, UK, 2017

(ArA) Arthurs A. M., Complementary Variational Principles, Clarendon Press, Oxford, 1970

(BlS), Blackburn S., The Big Questions, Philosophy, Quercus Publishing Plc., 2009, "What fills up space? The curious nature of things and their properties“

(DeR) Dedekind R., Continuity and Irrational Numbers, Essays on the Theory of Numbers, Continuity and Irrational Numbers, Dover Publications, New York

(DiR) Dicke R. H., Gravitation without a Principle of Equivalence, Rev. Mod. Phys. 29, 1957, pp. 363-376

(EiA) Einstein A., Grundzüge der Relativitätstheorie, Vieweg & Sohn, Braunschweig, Wiesbaden, 1992

(EiA1) Mein Weltbild, Ullstein Verlag, 2019

(EiA2) Einstein A., Ether and the theory of relativity, an address delivered on May 5th, 1920, in the University of Leyden

(FeE) Fermi E., Quantum Theory for Radiation, Reviews of Modern Physics, Vol. 4, 1932

(FlH) Flanders H., Differential forms with applications tot he physical sciences, Dover Publications, Inc., New York, 1989

(HeM) Sein und Zeit, Max Niemeyer Verlag, Tübingen, 2001

(HeW) Heisenberg W., Physikalische Prinzipien der Quantentheorie, Wissenschaftsverlag, Mannhein, Wien, Zürich, 1991

(KaI) Kant, I, The Critique of Pure Reason, Translated by J. M. D. Meiklejohn, ISBN-13: 978-1977857477

(KaM) Kaku M., Introduction to Superstrings and M-Theory, Springer Verlag, 1988, 1999

(KnA) Kneser A., Das Prinzip der kleinsten Wirkung von Leibniz bis zur Gegenwart, Teubner Verlag, Leipzig, 1928

(MiH) Minkowski H., Geometrie der Zahlen, Leipzig, Berlin, B. G. Teubner, 1910

(NaT) Nagel Th., Mind and Cosmos: Why The Materialist Neo-Darwinian Conception of Nature is Almost Certainly False, Oxford University Press, 2012

(PeR) Penrose R., The Large, the Small and the Human Mind, Cambridge University Press, 1997

(PeR1) Penrose R., The Emperor's New Mind: Concerning Computers, Minds, and the Laws of Physics, Oxford Univ. Press, 1989

(PoP) Poluyan P. V., Non-Standard Analysis of Non-classical Motion; do the hyperreal numbers exist in the Quantum-relative universe?

(ReW) Reijen van, W., Martin Heidegger, UTB, 2009

(ReW1) Reijen van, W., Heideggers ontologische Differenz, DZPhil, Berlin 52 (2004) 4, 519-539

(RiB) Riemann B., Über die Hypothesen, welche der Geometrie zugrunde liegen

(RoC) Rovelli C., Quantum Gravity, Cambridge University Press, Cambridge, 2004

(RoJ) Roberts J. T., Leibniz on Force and Absolute Motion, Philosophy of Science, Vol 70, No 3, pp. 553-573, 2003

(RuB) Russel B., The Philosophy of Leibniz, Routledge, London, New York, paperback edition, 1992

(ScA) Schopenhauer A., The world as will and representation, 1911

(ScE) Schrödinger E., Mein Leben, meine Weltansicht, dtv, 2006, My View of the World, Englisch translation: Cambridge University Press, 1964

(ScE1) Schrödinger E., What is Life? The Physical Aspect of the Living Cell with Mind and Matter, Cambridge University Press, 1967

(ScE2) Schrödinger E., Nature and the Greeks, Science and Humanism, Cambridge University Press, 1996

(ScE3) Schrödinger e., Space-Time Structure, Cambridge University Press, Cambridge, New York, Port Chester, Melbourne, Sydney, 1950,

(SchE) The Concept of Manifold, 1850-1950, Fachbereich Mathematik, Bergische Universitat Gesamthochschule Wuppertal, Gausstrasse 20, 42097 Wuppertal, Germany

(SchE1) Scholz E., Riemanns frühe Notizen zum Mannigfaltigkeitsbegriff und zu den Grundlagen der Geometrie, Archive for History of Exact Sciences, Vol. 27, No. 3 (1982), pp. 213-232

(SchE2) Scholz E., Geschichte des Mannigfaltigkeitsbegriffes von Riemann bis Poincaré, Birkhäuser, Boston, Basel, Stuttgart, 1980

(SmL) Smolin L., Einstein's Unfinished Revolution, The search for what lies beyond the quantum, Penguin Random House, UK, 2019

(StR) Streater R. F., Wightman A. S. PCT, Spin & Statistics, and all that, W. A. Benjamin, Inc., New York, Amsterdam, 1964

(SuL) Susskind L., Friedman A., Special relativity and classical field theory, Basic Books, New York, 2017

(TaR) Taschner R., Musil, Gödel, Wittgenstein und das Unendliche, Wiener Vorlesungen, Picus Verlag Wien, Bd. 87, 2002

(ToA) Topakkaya A., Die Grundzüge der Philosophie Henri Bergsons, GRIN Verlag GmbH, 2009

(UnA) Unzicker A., Einstein's Lost Key: How We Overlooked the Best Idea of the 20th Century, copyright 2015, Alexander Unzicker

(UnA1) Unzicker A., The Mathematical Reality, Why Space and Time are an Illusion, copyright 2020, Alexander Unzicker

(UnA2) Unzicker A., Jones S., Bankrupting Physics, How today's top scientists are gambling away their credibility, e-book; Unzicker A. Vom Urknall zum Durchknall, Springer Spektrum, Berlin, Heidelberg, 2010

(VeW) Velte W., Direkte Methoden der Variationsrechnung, B. G. Teubner, Stuttgart, 1976

(WeH) Weyl H. Was ist Materie, Verlag Julius Springer, Berlin, 1924

(WeH1) Weyl H., Philosophy of Mathematics and Natural Science

(WeH2) Weyl H., The Continuum, A Critical Examination of the Foundation of Analysis, Dover Publications, Inc., New York, 1994

(WiL) Wittgenstein L., Tractatus Logico-Philosophicus, Routledge, London, New York, 1992

(ZiR) Zimmer R. Arthur Schopenhauer, Ein philosophischer Weltbürger, dtv, München, 2012

Appendix I

Extract from (KaM)

Introduction to Superstrings and M-Theory

M. Kaku

1.2 Historical Review of Gauge Theory

(KaM): „Over the years, several proposals have been made to drop some of our commonsense about the Universe:

(1) Continuity

This approach assumes that space-time must be granular. The size of these grains would provide a natural cutoff for the Feynman integrals, allowing us to have a finite S-matrix. Integrals like

$$\int_{\varepsilon}^{\infty} d^4x$$

would then diverge as ε^{-n}, but we would never take the limit as ε goes to zero. Lattice gravity theories are of this type. In Regge calculus, for example, we latticize Riemannian space with discrete four-simplexes and replace the curvature tensor by the angular deficit calculated when moving in a circle around the simplex:

$$-\frac{1}{2\mu^2}\sqrt{-g}R \rightarrow angular\ deficit.$$

At present, however, there is no experimental evidence to support the idea that space-time is granular. Although we can never rule out this approach, it seems to run counter to the natural progression of particle physics, which has been to postulate larger and more elegant groups"

(2) Causality

This approach allows small violations in causality. Theories that incorporate the Lee-Wick mechanism are actually renormalizable, but permit small deviations from causality. These theories make the Feynman diagrams converge by adding a fictitious Pauli-Villars field of a mass M that changes the ultraviolet behavior of the propagator. … This means that the theory will be riddled with negative probabilities. …. that is, you can meet your parents before you are born

(3) Unitarity

We can replace Einstein's theory, which is based on the curvature tensor, with a conformal theory based on the Weyl tensor by conformal tensor based on the Weyl tensor:

$$\sqrt{-g}R_{ij}g^{ij} \rightarrow \sqrt{-g}C^2_{ij\rho\sigma}.$$

…. the conformal tensor posesses a larger symmetry group than the curvature tensor, that is invariant under local conformal transformations. … The Weyl theory is a higher derivative theory. …. The most optimistic scenario would be to have these unitary ghosts „confined" by a mechanism similar to quark confinement.

(4) Locality

Over the years, there have also been proposals to abandon some of the important postulates of quantum mechanics, such that locality. After all, there is no guarantee that the laws of quantum mechanics should hold down to distances of 10^{-33} cm. However, there have always been problems whenever physicists tried to deviate from the laws of quantum mechanics, such that causality. At present, there is no successful alternative to quantum mechanics.

(5) Point Particles

Finally, there is the approach of superstrings, which abandons the concept of idealized point particles. … "

Extract from (WeH3)

Philosophy of Mathematics and Natural Science

H. Weyl

The Physical Picture of the World

B. Matter and Fields. Ether

p. 171: „Just as the velocity of a water wave is not a substantial but a phase velocity, so the velocity with which an electron moves is only the velocity of an ideal „center of energy", constructed out of the field distribution. According to this view, there exists but one kind of natural laws, namely, field laws of the same transparent nature as Maxwell had established for the electromagnetic field. The obscure problem of laws of interaction between matter and field does not arise. This conception of the world can hardly be described as dynamical any more, since the field is neither generated nor acting upon an agent separate from the field, but following its own laws is in a quiet continuous flow. It is of the essence of the continuum. Even the atomic nuclei and the electrons are not ultimate unchangeable elements that are pushed back and forth by natural forces acting upon them, but they are themselves spread out continuously and are subject to fine fluent changes.

On the basis of rather convincing general considerations, G. Mie in 1912 pointed out a way of modifying the Maxwell equations in such manner that they might possibly solve the problem of matter, by explaining why the field possesses a „granular" structure and why the knots of energy remain intact in spite of the back and forth flux of energy and momentum. The Maxwell equations will not do because they imply that the negative charges compressed in an electron explode; to guarantee their coherence in spite of Coulomb's repulsive forces was the only service still required of substance by H. A. Lorentz's theory of electrons. The preservation of the energy knots must result from the fact that the modified field laws admit only of one state of field equilibrium – or of a few between which there is no continuous transition (static, spherically symmetry solutions of the field equations). The field laws should thus permit us to compute in advance charge and mass of the electron and the atomic weights of the various chemical elements in existence. And the same fact, rather than the contrast of substance and field, would be the reason why we may decompose the energy or inert mass of a compond body (approximately) into the non-resolvable energy or its last elementary constituents and the resolvable energy of their mutual bond."

Appendix C:
Quantum Physics and Causality

p. 258: „Quantum physics does not force a discontinuous time upon us even if the number of quantum states separable by a grating is universally limited. During the infinitesimal time interval dt the vector space experiences a certain infinitesimal rotation imparting the increment $\overrightarrow{dx} = L\vec{x} \cdot dt$ to the arbitrary vector $\vec{x}$. This dynamical law $\overrightarrow{dx}/dt = L\vec{x}$ (in which the operation L is independent of t and $\vec{x}$) is expressed in terms of Cartesian coordinates x_i by equations of the form

$$\frac{dx_i}{dt} = \sum_J l_{ij}\, x_j(t),\ i, j = 1, \dots, n$$

with given constant antisymmetric coefficients l_{ij} ($l_{ij} = -l_{ij}$). The salient point is that the wave state $\vec{x}$ varies according to a strict causal law; its mathematical simplicity is gratifying. A grating $G = \{E_1, \dots E_r\}$ and the corresponding quantum states $(G;1), \dots (G;r)$, are stationary if the subspaces $\boldsymbol{E}_i$ are invariant in time, i.e. if the linear operators E_i commute with the linear operator L.

*p. 259: „A system is never completely isolated from its surroundings, and its wave state is therefore subject to perpetual disturbances. This is the reason why the **secondary statistics of thermodynamics** is to be superimposed upon the **primary statistics** dealing with a given **wave state and its reaction to a grating**."*

p. 260: „The description here given must be corrected throughout in one point: the coordinates x_j in the underlying n-dimensional vector space are not real but arbitrary complex numbers and as such have an absolute value $|\vec{x}|$ and a phase. The square of the length of the vector is expressed in terms of the absolute values of the coordinates. The simplest of all dynamical laws $\overrightarrow{dx}/dt = L\vec{x}$ in such a complex space is of the form

$$(*) \quad \overrightarrow{dx}/dt = i \cdot \nu \cdot \vec{x}.$$

Here ν is a real constant. The wave state $\vec{x}$ then carries out a simple oscillation of frequency

$$\vec{x} = \overrightarrow{x_0}\{\cos(\nu t) + i sin(\nu t)\}, \quad \overrightarrow{x_0} = const.,$$

and hence the energy has the definite constant value νh (Planck's law). But whatever the dynamical law $\overrightarrow{dx}/dt = L\vec{x}$, the space can always be broken up into a number of mutually orthogonal subspaces $\boldsymbol{E}_j$ ($j = 1, .., r$) such that an equation (*) with a definite frequency $\nu = \nu_j$ holds in $\boldsymbol{E}_j$. The grating $G = \{E_1, \dots E_r\}$ thus obtained is stationary *and effects a sifting with respect to different frequencies* ν_j *and corresponding energy levels* $U_j = h\,\nu_j$. *Thermodynamics is based on this* G. *Any vector* $\vec{x}$ *in* $\mathbf{E}_j$ *satisfies the equation* $L\vec{x} = i\nu\vec{x}$ $(\nu = \nu_j)$*, and this fact is expressed in mathematical language by saying that* $\vec{x}$ *is an eigenvector of the operation* L *with the eigenvalue* $i\nu$. *The operator* $H = \frac{h}{i}L$*, called energy, has the same eigenvectors, but the corresponding eigenvalues are the energy levels* $= h\nu$. *The general equation* $\overrightarrow{dx}/dt = L\vec{x}$ *now reads*

$$\frac{h}{i}\frac{\overrightarrow{dx}}{dt} = H\vec{x} \quad \text{(Schrödinger's equation).“}$$

Appendix E:
Physics and Biology

topic „virus“

p. 276: „Incidentally, the gap between organic and inorganic matter has been bridged to a certain extent by the discovery of viruses. Viruses are submicroscopic entities that behave like dead inert matter unless placed in certain living cells. As parasites in these cells, however, they show the fundamental chracteristics of life – self-duplication and mutation. On the other hand many viruses have the structure typical of inorganic matter; they are crystals. In size they range from the more complex protein molecules tot he smaller bacteria. Chemically they consist of nucleo-protein, as the genus do. A virus is clearly something like a naked gene. The best studied virus, that of tobacco mosaic disease, is a nucleo-protein of high molecular weight consisting of 95 per cent protein and 5 per cent nucleic acid; it cristallizes in long thin needles.“

p. 277: „The specific properties of living matter will have to be studied within the general laws valid for all matter; the viewpoint of holism that the theory of life comes first and that one descends from there sort of deprivation to inorganic matter must be rejected. It is therefore significant that certein simple and clearcut traits of wholeness, organization, acausality, are ascribed by quantum mechanics to the elementary constituents of all matter.“

p. 277: „The quantum physics of atomic processes will become relevant for biology wherever in the life cycle of an organism a moderate number of atoms exercises a steering effect upon the large scale happenings. …. On a broad empirical foundation, genetics furnishes the most convincing proof that organisms are controlled by processes of atomic range, where the acausality of quantum mechanics may make itself felt. … The mere fact of such X-rays induced mutations proves that the genes are physical structures.“

p. 278: „By ingenious methods H. J. Muller, N. W. Timoféeff-Ressowsky, and others have succeeded in establishing simple quantitive laws concerning the rate of induced mutations. These results indicate that the mutation is brought about by a single hit, not by the concerted action of several hits, and that this hit consists of an ionization, and is not, as one might have thought, a process directly released by the X-ray photon or absorbing the whole energy of the secondary electron.“

These facts suggest the hypothesis that a gene is a (nucleo-protein) molecule of highly complicated structure, that a mutation consists in a chemical change of this molecule brought about by the effect of an ionization on the bonding electrons, and that thus allele genes are essentially isometric molecules.“

The observed absolute rate of mutations would be explained if a specifc mutation requires that a hit occurs within a critical volume (‚target‘) in the gene, the magnitude of which amounts to about 5-10 A cube (5-10 atomic distances cube). The physicist finds it, if not plausible at least acceptable, that a quantum jump at a specific point requiring an activation energy of about 1.5 is relased by a hit of 30 electron volts within a sensitive volume of 5-10 A cube. The observed thermic variation of the spontaneous mutation rate (van't Hoff's factor) is in good quantitative agreement with the picture.“

Extract from (DeR1)

Continuity and Irrational Numbers

R. Dedekind

§3 Continuity of the straight line

*„Of the greatest importance, however, is the fact that in the straight line L there are infinitely many points which correspond ton o rational number. …. The straight line L is infinitely richer in **point-individuals** than the domain Q of rational numbers in **number-individuals**.*

*If now, as is our desire, we try to follow up arithmetically all phenomena in the straight line, the domain of rational numbers is insufficient and it becomes absolutely necessary that the instrument Q, constructed by the creation of rational numbers be essentially improved by the creation of new numbers such that the domain of numbers shall gain the same completeness, or as we may say at once, the same **continuity**, as the straight line.*

… For the way in which the irrational numbers are usually introduced is based directly upon the conception of extensive magnitudes – which itself is nowhere carefully defined – and explains number as the result of measuring such magnitude by another of the same kind. Instead of this I demand that arithmetic shall be developed out of itelf. … Just as negative and fractional rational numbers are formed by a new creation, and as the laws of operating with these numbers must and can be reduced tot he laws of operating with positive integers, so we must endeavor completely to define irrational numbers by means of the rational numbers alone. The question only remains how to do this.

I find the essence of continuity in the following principle:

> *„If all points of the straight line fall into two classes such than every point of the first class lies to the left of the second class, then there exists one and only one point which produces this division of all points into two classes, this severing of the straight line into two portions."*

*As already said I think I shall not err in assuming that every one will at once grant the truth of this statement; the majority of my readers will be very much disappointed in learning that this common-place remark the secret of continuity is to be revealed. To this I may say that I am glad if every one finds the above principle so obvious and so harmony with his his own idea of a line; for I utterly unable to adduce any proof of its correctness, nor has anyone the power. The assumption of this property of the line is nothing else than an axiom by which we attribute to the line its continuity, by which we find continuity in the line. If space has at all a real existence it is **not** necessary for it to be continuous; many of its properties would remain the same even were discontinuous. And if we knew for certain that space was discontinuous there would be nothing to prevent us, in case we so desire, from filling up ist gaps, in thought, and thus making it continuous; this filling up would consist in a creation of new point-individuals and would have to be effected in accordance with the above principle."*

Extract from (EbH)

Numbers

Ebbinghaus H.-D. et al.

Chapter 11, §2

The dimension of a divisional algebra and the sphere S^{n-1} is only parallelizable for $n = 1,2,4,8$

> *Theorem:*
>
> *If the mod-2 invariant of a continuous mapping $f: S^{n-1} \to GL(n)$ (GL(n) the topology group of nxn invertible matrices) is different from zero, than it is $n = 1,2,4,8$.*
>
> *Theorem:*
>
> *The sphere S^{n-1} is only parallelizable for dimensions $n = 1,2,4,8$; actually one should exclude the case $n = 1$ case, because it eventually leads to trivial additional considerations.*

We note that $U(n)$ (the group of $n \times n$ unitary matrices, which is a sub-group of the general linear group $Gl(n, C)$) is the semi-direct product of $U(1)$ with $SU(n)$, and $U(1) \cong S^1$ and $SU(2) \cong S^3$.

Appendix II

Summary
from the no longer maintained page:
quantum-gravitation@de

This summary is concerned with those philosophical concepts, which are in line with the mathematical axioms of the proposed gravity and quantum field model. The methodology for this kind of consistency check is about the „Form-Fit-Function" concept, i.e.

Form
The building of the form is essentially about a tbd common language based on adequately defined terms. It is built on the ideas and notions of Kant, Schopenhauer, Schrödinger in combination with the proposed mathematical concepts.

Fit
The „Fit" is achieved by consistency across the considered areas „philosophy", „philosophy of mathematics", and „philosophy of physics", targeting for a consistent (language) framework, "only", not for a "true" ideology / religion, which is anyway always only built on human judgements.

Function
is achieved by integrating compatible concepts (e.g. teleology, onenes of mind, continuum, monads, infinitesimal "matter contact transforms" w/o only affine (momentum) directions, non experienced "be-ing" from the above (form) areas.

Example: Leibniz' least action principle

Leibniz's basic conception is about the fact, that natural processes can be derived from (science specific) integral principles. A general teleology is about the fact that for every perception view (caused by a physical event) there can be detected a corresponding effort principle.

The principle of least action is the (purely) **form** of the (Leibniz) integral principle. It is characterized by the fact that the present is determined by the past and the future, while the corresponding natural principle definition of Newton determines the future by the past and the present.

Related to the purely *form* there are multiple perception area /sciences specific integral principles.

(KnA) p. 43, "*Den tiefsten Zusammenhang der Teleologie oder sagen wir geradezu des Prinzips der kleinsten Wirkung mit der Kantischen Gedankenwelt gewinnen wir erst, wenn wir uns der Kritik der Urteilskraft zuwenden* ..."
(KnA) p. 55, " ... *so dürfen wir endgültig als Beziehung unseres Prinzips zur Kantischen Urteilskraft feststellen: D a s P r i n z i p d e r k l e i n s t e n W i r k u n g i n s e i n e r m o d e r n s t e n A l l g e m e i n h e i t i s t e i n e M a x i m e d e r r e f l e k t i e r e n d e n U r t e i l s k r a f t*."

CONTINUUM, MATTER, FORM, SUBSTANCE & MATHEMATICAL CONTINUITY

The physical concept of a point particle charge (Dirac), which is required to test the presence of (continuously "acting") forces, is the root cause for current conceptual miss matches between quantum and gravitation theory. For a gravity field, as well as for a quantum field point wise convergence of functions is of no interest. A mathematical GUT model needs to overcome the corresponding inherited constraints, basically caused by the concept of physical "point particles", which goes along with the requirement to formulate a ("continuous") contact transformation (S. Lie) between "objects" w/o extensions.

THE SCOPE IN A NUTSHELL

Schrödinger	oneness mind/psyche	*effective operations / differential change*
Schrödinger	oneness form/shape	*no substance & no observations geom. shape*
Kant	teleology	*proposed Nature* effectiveness *principle*
Kant	judgments	*verification of judgment&reality=(again judgm.*
Schrödinger	organism/metabolism	*insistence: form/shape vs. not minutiue: material*
Schopenhauer	anorganic matter	*insistence: material vs. not minutiue: form/shape*
Schopenhauer	opposition	*both insistence/not minutiue: judgment of subject*
Schrödinger	fractale/discrete	*judgm. change=discrete, even i. perfect continuum*
Heidegger	The(re-) being	*new* ontological *difference principle*
Heidegger	"The turn"	*mind & form: two sides of the same coin*.

„FIT FOR PURPOSE"

THOUGHTS FROM SCHRÖDINGER, HEIDEGGER, SCHOPENHAUER & KANT

Schrödinger

Schrödinger emphasis e.g. on the question, when starting from a particle, then atome, molecule the chain up to an organism the entity starts to exist: a cell division generates two entities, just this and this is a discrete phenomenon. The counterpart in the plant life (flora) is the photosynthesis, which is perceived as a distant effect phenomenon, when solar energy is transformed to chemical energy. Both "development" processes generate a kind of *There-being*.

E. Schrödinger, (ScE1)
"The objectice world has only been constructed at the prize of taking the self, that is, mind, out of it remaking it; mind is not part of it; obviously, therefore, it can neither act on it nor be acted on by any of its parts. If this problem of the action of mind on matter cannot be solved within the framework of our scientific representation of the objective world, where and how can it be solved?"

"No single man can make a distinction between the realm of his perceptions and the realm of things that cause it, since however detailed the knowledge he may have acquired about the whole world, the story is occuring only once and not twice. The duplication is an allegory suggested mainly by communication with other beings."

(ScE2) pp. 115-151:
"*Radical change in our ideas of matter*"
"*Form, not substance, the fundamental concept"*
"*The nature of our 'models'* "
"*Continuous description and causality*"
"*The intricacy of the continuum (das Kontinuumsproblem)"*
"*The alleged break-down of the barrier between subject and object*"

(ScE1): *"Vielleicht ist es sogar zulässig zu sagen: Metaphysik v e r w a n d e l t sich im Laufe der Entwicklung in Physik - freilich nicht in dem Sinne, wie es v o r Kant den Anschein haben mochte. Nämlich n i e durch allmähliche Sicherstellung vorerst noch unsicherer Meinung, sondern durch Klärung und W e c h s e l des philosophischen Standpunktes."*

(ScE2), p. 31 "*Die Substanz hat ihre Rolle ausgespielt. Wir haben es nur mit Gestalten zu tun, die teils wechseln, aber doch auch verharren. .... Dabei müssen wir freilich Gestalt in viel weiterem sinn verstehen denn geometrische Form. Es gibt überhaupt keine Beobachtung, die auf die räumliche Gestalt eines Partikels oder eines Atoms abzielte*."

(ScE2)*, p. 125, "But when you come to the ultimate particles constituting matter, there seems to be no point in thinking of them again as constituting of some material. There are, as it were, purely shape, nothing but shape; what turns up again and again in successive observations is this shape, not an individual speck of material.*

In this we must, of course, take shape (or Gestalt) in a much wider sense than as geometrical shape. ***Indeed there is no observation concerned with the geometrical shape of a particle or even with an atom****."*

Schrödinger's major critique is about the common handicap of all western philosophy baseline assumptions, which is about *spacial and temporal multiplicity of examining and thinking individuals*. He is just rejecting this multiplicity and proposes instead a purely monoism of psyche ("cogitat - est") with its two parts, consciousness and subconsciousness, refering to Buddhistic philosophy (philosophy of Vedanta): "*multiplicity is only a p p e a r a n c e s, i t d o e s n o t e x i s t i n r e a l i t y*". The later one (subconsciousness) ensure functional / proper operations of this world's organisms (relating to the *"that which has being*"), while the first one is triggered, when differential changes happen to "effective operations" (of organisms); this is related to the philosophical term "das *Werdende*".

While Schrödinger refers back to philosophy history proposing new ideas with respect to the "*mind & matter*" question, Weyl refers back to philosophy with respect to the concept of "*infinity*". (The concept of a "zero" was introduced when changing from roman numbering system to decimal system to enable calculus dealing only with 10 integers, instead of infinitely integers).

(ScE1) p. 119, "*The second (antinomy) is our fruitless quest fot he place where mind acts on matter or vice-versa, so well known from Sir Charles Sherrington's honest search, …in "Man on his Nature". The material world has only been constructed at the prize of taking the self, that is, mind, out if it, removing it; mind is not part of it; obviously, therefore, it can neither act on it be acted on by any of its parts.* " ..

(ScE1) p. 121,"*In my own words I would express this by saying: Mind has erected the objective outside world of the natural philosopher out of its own stuff. Mind could not cope with this gigantic tasks otherwise than by the simplifying device of excluding itself – withdrawing from its conceptual creation. Hence the latter does not contain its creator. …*

Physical science … faces us with the impasse that mind per se cannot play the piano – mind per se cannot move a finger of a hand. Then the impasse meets us. The blank of the "how" of mind's leverage on matter. The *inconsequence staggers us. Is it a misunderstanding?"* ..

(ScE1) p. 122, *"Neither can the body determine the mind to think, nor the mind determine the body to motion or rest or anything else (if such there be)."*

(ScE1) *p. 128, "The reason why our sentient, percipient and thinking ego is met nowhere within our scientific world picture can easily be indicated in seven words:* ***because it is itself that world picture****."*

Weyl / Schrödinger

Matter, mind, mathematics and natural science

(ScE) p. 49, "*Vielleicht ist es sogar zulässig zu sagen: Metaphysik v e r w a n d e l t sich im Laufe der Entwicklung in Physik - freilich nicht in dem Sinne, wie es v o r Kant den Anschein haben mochte. Nämlich e i n e durch allmähliche Sicherstellung vorerst noch unsicherer Meinung, sondern durch Klärung und W e c h s e l des philosophischen Standpunktes."*

(NaT) p. 13: "*Ausgangspunkt für meine Argumentation ist das Scheitern des psychophysischen Reduktionismus, eine Position in der Philosophie des Geistes, die weitgehend von der Erwartung motiviert ist, zeigen zu können, dass die physikalischen Wissenschaften im Prinzip eine Theorie von allem liefern könnten*"

(TaR) p. 31, "*Gödel bewies: Cantor, der sich .... bemühte, die Stufe der Unendlichkeit des Kontinuums zu fixieren, musste scheitern. Steckt man die Mengenlehre in ein formales Korsett, kann niemand, ..., die Unendlichkeit des Kontinuums orten"*

(TaR) p. 32: *"… dass aber gerade die naheliegende Frage nach dem Wesen der Unendlichkeit einer Geraden unentscheidbar sein sollte, empfand er (*Gödel*) trotzdem als kläglichen Mangel der Mengentheorie"*

(ReW) p 12, "nur Philosophie kann zeigen, warum die Erkenntnisse er Wissenschaften wirklich wahr sind, sie liefert die Kriterien mit Hilfe derer wir zwischen Wahrheit und Unwahrheit unserer Urteile unterscheiden können."

(ReW) p. 17, "*Sein ist das transcendens schlechthin"*

(ReW) p. 19, "*Heidegger versucht mit dem Denken des Daseins, die Trennung von res extensa und res cogitans ... zu unterlaufen. ..... bekämpft Heidegger die nach ihm auf Descartes zurückgehende Vorstellung, dass das Subjekt als die Grundlage und gar das Zentrum der Philosophie gedacht werden sollte. Wer so denkt, verfehle unvermeidlich die Frage nach dem Sein und dem Dasein, weil er beide ontisch denkt.... Das, was Heidegger ontologische Differenz nennt, ist also der Unterschied von Sein und Seiendem, zugleich aber ihre dynamisch gedachte Identität. ...."*

(ReW) p. 109, "*Heidegger: Sprache ist das Haus des Seins*"

(WeH) p. 18: *"I am convinced that the substance has lost its role in physics"*

(WeH) p. 19: ""*the concept of "momentum" appears to be primarily to the concept of "mass/matter""*

(WeH) p. 20. *"the mass of a body is determined by its state"*

(WeH) p 31: *"when using a test particle to test/model the action of a field one already disturbs the field"*

(WeH) p. 44: *"a strictly intuitive rational of a mathematical theory of the continuum (as drafted by Brouwer and Weyl) were required to build the continuum as a medium, where single particles can be identified, but where the set of particles can be resolved"*

(WeH) p. 49: *"the today's relationship between matter and field is dynamical: the matter builds the field, the field acts on the matter*"

(WeH) p. *"For Leibniz the "reality" of movement is not built on movements (change of the position), but on the causing force; "La substance est un etre capable d'action - une force primitive""*

(WeH) p. 58: *"....the Leibniz agens theory of matter can be executed by the GRT. Based on this a matter particle is even not a point in the field space, even not any kind of something related to "space" (extensive)"*

(WeH) p. 59: *"what is matter? After the perception of the concept of substance has been quashed, the today's beam vacillates between a dynamic and a field theory of matter"*

(BlS)… Kant thought, that if we can only know objects because of their potential effects on others, their powers, then it seems that we are only responsive to what they do but not responsive, necessarily, to what they are. He thought that there have to be „other intrinsic properties, without which the relational properties would not exist because there would be no subject in which they inhered". But it is not clear how we can know

about this „subject"…. Are we therefore cut off from the world as that? Then we would be caught in a „false imaginary world" (Bishop Berkeley).

Michael Faraday thought, that we could just do without Kant´s „other intrinsic properties". Suppose we try to distinguish a particle x from the powers or forces m whereby it makes its influence known. Then, Faraday writes,

„"to my mind … the x or nucleus vanishes, and the substance consists of the power, or m, and indeed what notion can we form of the nucleus independent of its power: what thought remains on which to hang the imagination of an x independent of the acknowledged forces? Why then assume the existence of that of which we are ignorant, which we cannot conceive, and for which there is no philosophical necessity?""

The problem which this is whether we can be satisfied with the idea that „the substance consists of the powers", or whether contrary to Faraday there is some kind of philosophical necessity to posit a substance as well, a nucleus or thing that actually possesses the powers.

But there is an argument that we need Kant´s further category of intrinsic properties. We might call it the not-just-washing argument, after Bertrand Russell, who talks in his book „The Analysis of Matter" of how „there are many possible ways of turning things hitherto regarded as „real" into mere laws concerning the other things," and remarks, „Obviously there must be a limit to this process, or else all things in the world will merely be each other´s washing." The conclusion is that even if we have trouble understanding things apart from their powers, nevertheless we seem to need them. We seem to need them because otherwise we have no conception at all the actual world.

(ScE1) p. 1594):

"***... a truly infinitesimal geometry ...should know a transfer principle for length measurements between infinitely close points only***".

(TaR) p. 17, *„Das Problem, von dem die Rede ist, lautet knapp formuliert so: Wie hängen Arithmetik und Geometrie, die beiden Grundpfeiler der Mathematik, zusammen? …*

***Geometrie**, …, fußt unmittelbar auf sinnlichen visuellen Eindrücken, raubt ihnen jedoch Buntheit, Körperlichkeit, Vergänglichkeit, Verletzbarkeit, kurz: alle opaken und barocken Reize. Geometrie verkürzt die optische Wahrnehmung so lange, bis nur mehr einzelne Punkte und Linien übrig bleiben. …*

***Arithmetik**, die Theorie der Zahlen, …, hat hingegen nur mittelbar mit sinnlichen Eindrücken zu tun: zwar sehen und fühlen wir Geldstücke, die wir zählen, aber wir sehen und fühlen nicht die Zahl Dreißig selbst, sondern nur die Silberlinge, die wir mental mit Dreißig verbinden. ….Weder optisch, noch taktil, noch akustisch, noch sonst wie sinnlich empfinden wir eine Zahl sui generis, kein Sinnesorgan vermag sie direkt zu empfangen.*

Jedoch: Man kann Zahlen geometrisch veranschaulichen. Jeder Maßstab beleg es: Er bringt Zahlen auf einer Gerade unter. Arithmetik erweist sich folglich als geometrische Disziplin. Wie man hingegen alle, ausnahmslos alle Punkte einer Geraden umgekehrt als Zahlen zu deuten vermag, blieb seit den Tagen des Pythagoras … ein Rätsel.

… Ist die Geometrie sogar so exakt, dass sich die sinnlichen Anschauungen, auf der sie zu beruhen scheint, in Wahrheit als überflüssig entpuppt? Würden wir, ohne Rückgriff auf Sehen und Tasten, alleine aufgrund arithmetischer Gesetze alle Einsichten und Erkenntnisse der Geometer gewinnen?"

(ScE1) p. 77: " *Der Grund dafür, daß unser fühlendes, wahrnehmendes und denkendes Ich in unserem naturwissenschaftlichen Weltbild nirgends auftritt, kann leicht in fünf Worten ausgedrückt werden: Es ist selbst das Weltbild. ... Aus diesem (*arithmetischen*) Paradoxon gibt es zwei Auswege, die beide vom Standpunkt unseres heutigen naturwissenschaftlichen Denkens aus .... reichlich unsinnig aussehen. Der eine ist die Vervielfachung der Welt in Leibniz' schrecklicher Monadenlehre, in der jede Monade eine Welt für sich ist, es ist keine Verbindung zwischen ihnen. .... Offenbar gibt es nur e i n e n andren Ausweg: die Vereinigung aller Bewußtseine in eines. Die Vielheit ist bloßer Schein; in Wahrheit gibt es nur ein Bewußtsein.*

(ScE1) p. 109: "*Indessen liegt die überragende Bedeutung von Kants Behauptung gar nicht in einer richtigen Verteilung der Rollen auf den Geist und auf sein Objekt -die Welt- in dem Prozeß, in dem "sich der Geist eine Vorstellung von der Welt bildet". Das Große war, den Gedanken zu fassen, daß dieses e i n e D i n g - Geist oder Welt - sehr wohl andrer Erscheinungsformen fähig sein kann, die wir nicht zu erfassen vermögen und die die Begriffe Raum und Zeit nicht enthalten. Das bedeutet eine eindrucksvolle Befreiung von einem eingewurzelten Vorurteil. Wahrscheinlich gibt es andre Arten, die Erscheinungswelt zu ordnen als die raum-zeitliche. Ich glaube, es war* Schopenhauer*, der Kant zuerst so verstanden hat."*

(ScE1) p. 117: "*Einstein hat nicht - ... - Kants tiefe Gedanken über die Idealisierung von Raum und Zeit widerlegt. Er hat im Gegenteil einen großen Schritt in Richtung auf ihre Vollendung gemacht.*

(ScA) Bd 2, §29, 193: *"Daß diese Welt, in der wir leben und sind, ihrem ganzen Wesen nach, durch und durch Wille und zugleich durch und durch Vorstellung ist; daß diese Vorstellung schon als solche eine Form voraussetzt, nämlich Objekt und Subjekt, mithin relativ ist; und wenn wir fragen, was nach Aufhebung dieser Form und aller ihr untergeordneten, die der Satz vom zureichenden Grund ausdrückt, noch übrig bleibt; dieses* *als ein von der Vorstellung* toto genere *Verschiedenes, nichts Anderes seyn kann, als Wille, der sonach das eigentliche Ding an sich ist. Jeder findet sich selbst als diesen Willen, in welchem das innere Wesen der Welt besteht, so wie er sich auch als das erkennende Subjekt findet, dessen Vorstellung die ganze Welt ist, welche insofern nur in Bezug auf sein Bewußtseyn, als ihrem nothwendigen Träger, ein Daseyn hat. Jeder ist also in diesem doppelten Betracht die ganze Welt selbst, findet beide Seiten derselben ganz und vollständig in sich selbst. Und was er so als sein eigenes Wesen erkennt, das Selbe erschöpft auch das Wesen der ganzen Welt, des Makrokosmos: auch sie also ist, wie er selbst, durch und durch Wille, und durch und durch Vorstellung, und nichts bleibt übrig. So sehn wir hier die Philosophie des Thales, die den Makrokosmos, und die des Sokrates, die den Mikrokosmos betrachtete, zusammenfallen, indem das Objekt beider sich als das Selbe aufweist."*

Kant

"Critique of Judgment"

(KnA) p. 43, "*Den tiefsten Zusammenhang der Teleologie oder sagen wir geradezu des Prinzips der kleinsten Wirkung mit der Kantischen Gedankenwelt gewinnen wir erst, wenn wir uns der Kritik der Urteilskraft zuwenden, und zwar unter der Führung des hervorragenden Neukantianers Stadler,* ..."

(KnA) p. 55, " ... *so dürfen wir endgültig als Beziehung unseres Prinzips zur Kantischen Urteilskraft feststellen: D a s P r i n z i p d e r k l e i n s t e n W i r k u n g i n s e i n e r m o d e r n s t e n A l l g e m e i n h e i t i s t e i n e M a x i m e d e r r e f l e k t i e r e n d e n U r t e i l s k r a f t*."

A big miss-understanding concerning mathematical model of acknowledged physical laws is, that 'real numbers' are all real, in fact nearly all of them are transcendental; the set of rational numbers is a zero set in the sense of the Lebesgue integral, which is the standard inner product for probability theory and quantum theory. As a consequence every mathematical ((weak or strong) partial differential equation(s)) model, based on which a physical law is described, is per definition transcendental.

Current inconsistency between the mathematical model of quantum theory and gravity theory are therefore inconsistent from a mathematical model point of view in the transcendental area. The root causes of this are either over-determined axioms/principles (put in place by human reason/mind) or inconsistent axioms, if one rejects the (stupid?) option, that there can be two or more transcendental areas, which are "inconsistent" from a mathematical point of view.

Each mathematical model built on real numbers is per definition transcendental; if the model is declared as a physical law, this is building of human reason/*Vernunft*, which goes beyond human intellect/mind/ understanding/*Verstand*.

According to ***Critique of Pure Reason*** (*the area, which human develops by sensuous perception; Nature / recognition / notion / sensualism; causality*) the intellect a priori disposes about notions which prove right but which cannot a posteriori be verified. The same approach is applied by Kant in his ***Critique of Practical Reason*** (*the area, which human develops only in human thoughts/mind; human /will / freedom / transcendence / supernatural; purpose*) to "explain", why there is obviously a room of freedom for human beings to decide, while everything else in the Nature seems to determine by causality only. Not the intellect defines a priori laws/axioms, but the reason. The area of the *Critique of Practical Reason* is per definition the transcendence, which is the area, which can be acknowledged by the human being only in his thoughts. The same is true for real numbers, as mentioned above.

We emphasis, that variational equations are equivalent (if certain regularity requirements are fulfilled) to corresponding "operator norm (action) minimization" problems. This is per definition a model following a principle of "purpose/effectiveness", not of causality.

The concept of "effectiveness" (teleology) to build the bridge between *Critique of Pure Reason* and *Critique of Practical Reason*, answering the "problem of the concrete", was introduced by Kant in his "***Critique of Judgment*** (CoJ)", (LuR), p. 121 "*Die teleologische Ureteilkraft*".

Quotes from (KaI)

"*Als* (innerer Natur-) *Zweck ist ein Ding nur möglich, wenn es nicht ursprünglich von einem Mechanismus der Natur verursacht ist, sondern von einer Wirkung, die durch Begriffe bestimmt ist. Auch darf seine Form nicht vom Verstand alleine erkannt werden können. ..*

Ich würde vorläufig sagen: das Ding existiert als Naturzweck, wenn es von sich selbst (obgleich in zwiefachem Sinne) Ursache und Wirkung ist.

Ein organisiertes Wesen ist also nicht bloß Maschine; denn die hat lediglich bewegende Kraft; sondern sie besitzt in sich bildende Kraft ..."

Kant's CoJ is about the subjective (aesthetical) and objective (teleological) concept of judgement is about the following structure:

Die teleologische Hypothese lautet, dass die Dinge vielleicht nicht alleine von wertfreier Chemie und Physik festgelegt werden, sondern außerdem noch von etwas anderem, nämlich einer kosmischen Prädisposition für die Schaffung von Leben, Bewusstsein und Wert, der von ihnen nicht ablösbar ist.

Der Hypothese einer natürlichen Teleologie zufolge besäße die Welt der Natur einen Hang, Wesen von der Art entstehen zu lassen, die ein Wohl haben -Wesen, für welche die Dinge gut oder schlecht sein können. Dies sind alle tatsächlichen und möglichen Lebensformen (Organismen).

Heidegger

Heidegger's concept of "*ontologische difference*" might provide the appropriate concept to link the philosophical terms of the-being / there-being with mathematical terms with respect to weak and strong PDO equations embedded in a distributional Hilbert scale framework.

(ReW1)
p. 519, *„Die ontologische Differenz ist … ein Grundbegriff. … Grundbegriffe …. Sind Begriffe, mit denen „der Grund" begriffen, gegriffen, gefaßt, ja überhaupt erst erreicht, ja zuvor erst auch nur erahnt werden soll". … selbstwidersprüchliche Moment bestimmt auch den Grundbegriff der ontologischen Differenz."*

p. 520, *„Der Unterschied kann jetzt nur so gedacht werden, dass er gleichzeitig als ein Unterschied ‚in' und ‚ausserhalb' des Daseins gedacht wird; wobei das ‚in' und ‚ausserhalb' gleichermassen als räumlich und zeitlich und als nicht-räumlich und nicht-zeitlich zu verstehen ist. Die logisch unzulässige Kombination von ‚innerhalb' und ‚ausserhalb" zeigt an, dass die Dimension der Alltagspraxis, für die das Gesetzt der Kausalität und des Widerspruchsverbots gilt, transzendiert wird*."

p. 522, *„Wissenschaftliche Erkenntnis scheint zunächst etwas zu sein, was …. in der empirischen Welt vorkommt – es gibt sie einfach: Sie ist ein Seiendes unter Seienden. Damit aber, dass dieses Seiende, weil es ja nicht nur ‚ist', sondern anderes Seiende objektivieren, zum Gegenstand machen kann und somit einen Unterschied zwischen sich und dem Objekt legen kann, zeigt sich, dass es etwas gibt, was diese Ebene des Seienden übersteigt – nennen wir es ‚Sein'."*

p. 523, *„Wir sollten Denken nicht nur als Feststellen von Qualitäten und Ursachen/Folgen verstehen, sondern vor allem als Fragen danach, wie das möglich ist." … Dieses Fragen ist ein Fragen nach der besonderen Art, in der die Wesen, die so fragen, ‚sind'. Es ist ein Fragen nach dem Sein des Seienden – also nach dem Unterschied von Sein und Seiendem. … Das heisst also, dass es das Sein nicht mehr gab, als es vergessen wurde.* ***Das Sein gibt es erst dann wieder, wenn es gedacht wird****.*

Wissenschaftliche Erkenntnisse werden in Urteilen gefasst. Die Übereinstimmung zwischen dem Urteil und der Wirklichkeit aber bleibt unsicher. Jede sogenannte Verifikation ist ja auch nichts weiter als wieder ein Urteil. …Jedes Urteil ist Interpretation, oder, wie Heidegger es nennt ‚Verstehen'."

p. 526, *„Wahrheit", so meint Heidegger, ist bei Plato und Aristoteles definitiv als Richtigkeit der Aussage (miss-) verstanden worden. … Das richtige Urteil beruht dabei auf einem angemessenen Vorstellen, das sich entweder – idealistisch – auf das vorgestellte (perceptum oder idea) oder auf den Gegenstand (Realismus) bezieht*."

p. 527, *„Heidegger fasst nun seinerseits die Beziehung zwischen Denken und Seiendem als ‚Offenheit'. Nur Dank der Offenheit ist Richtigkeit möglich - nicht umgekehrt*."

p. 529, *„Die Grundfrage richtet sich nun auf das ‚Kehrige' von Seyn und Seiendem, das sich im Dasein ereignet. Seyn und Seiendes werden nun als Gegensätze, die sich zu einander kehren, in einer Einheit gefasst. Sie sind dabei nicht als Teile eines Ganzes zu begreifen, denn sie sind zugleich Teil und ganzes, indem sie jeweils füreinander – und für sich – das Andere und das Selbe sind. .... Heidegger verabschiedet sich hier von seinem Ansatz in Sein und Zeit".*

p. 531, *„Es gibt also eine klar fassbare Korrespondenz zwischen Seyn und Da-sein*."

p. 537, "*Mit dem Denken der kehre setzte in Heideggers Denken eine radikale und konsequente Änderung ein. Mit ihm löst er erst die radikalen Versprechen aus Sein und Zeit ein."*

p. 538, *„Dieser Versuch artikuliert sich beispielhaft in der Auflösung der gängigen Vorstellungen von Identität und Differenz*."

(ReW) p. 32, "*Wissenschaftliche Erkenntnisse werden in Urteilen gefasst. Die Übereinstimmung zwischen dem Urteil und der Wirklichkeit aber bleibt unsicher. Jede so genannte Verifikation ist ja auch nichts weiter als wieder ein Urteil. Deswegen ist es ratsam, von vornherein zu akzeptieren, dass Urteile keinen absoluten Wahrheitsanspruch geltend machen können. Jedes Urteil ist Interpretation oder, wie Heidegger es nennt, "*Verstehen*"."*

Heidegger

"Science" and "Ontological Difference"

"*The "logical" concept of* ***science*** *understands science with respect to its results and defines it to be a "nexus of explanatory and true — i.e., valid, correctly formed — propositions". The existential concept of science understands it, in contrast, to be a mode of existence and therefore to be a mode of being-in-the-world, a mode that uncovers or discloses either what-is or being. A fully adequate existential interpretation of science can only be carried out once the sense of being and the relation between being and truth have been understood on the basis of the temporality of existence…It is only within this understanding that the ideal of phenomenology can be developed — in contradistinction to the pre-conception that has already been indicated.*

The essence of what today is called science is research. It is essential to research that cognition, as practice, orient itself toward a realm of beings — of nature or of history. Here, procedure does not mean just method or procedure; every practice has need of an open region within which it already moves. However, the opening up of such a region is the basic practice of research. The opening-up is carried out here by projecting within some realm of beings, e.g., within nature, a definite outline of nature-processes. The project predelineates in what way cognitive practice has to commit itself to the region which is opened up. It is this commitment or stricture which is the rigor of research. Through its projection of the outline and through the definition of rigor, the practice of research assures itself of its region of objects within the relevant realm of being.

Only on the way toward solving the basic ***ontological*** *problem of the* ***difference*** *and* ***relationship between being and beings*** *can the Kantian thesis "being is not a real predicate" be at once both grounded and complimented by a radical interpretation of being at large as extantness (actuality, being there, existence)."*

Penrose-Hawking

dispute about

"*The Nature of Space and Time*"

Penrose rep. Hawking are supporter of platonism resp. positivism. This goes along (more or less a kind of re-branding, building on the first Solvay conference) with the dualism of idealism resp. empiricism (F. Bacon). Both concepts one can belief or not, but it needs a choise. If one has made a choice, there were/are antonomies in both concepts. Kant was the first and last of the philosophers, who developped a concept to overcome this dualism. There is no chance to show/prove evidence to this concept, which is an intrinsic consequence of the concept of transcendence; and that's where we are and where we will be!

Penrose's "3-world-model" (PeR), which adds a "mind world" to "Plato's world" and "physical world", just increases the complexity of "Plato's world", while not adding any additional value to Kant's conception. Needless to say that Hawking's concept of integrating "mind world" into the "physical world" (i.e. finally mind can be "explained" experientially!) has also been overcome by Kant resp. is the source of antinomies.

Sorry for the kind of blasphemous sounding statement, but this comes across (just!) like a kind of reincarnation of Leibniz (mathematician) versus Newton (physicist), both "platonists", with their concepts of $E(pot)= m * v*v/2$ and (transcendental) monads versus $E(pot) = m* v * v$ (!!), $F = m * a$ and (trancendental (!)) particles.

The probably more fruitful approach to this "dualism" challenge could be an analysis of the question, which of the following alternatives are the more appropriate axiom in order to conceptual capture the "*problem of the concrete*" of the connection between the physical "object related/natural" relevance of physical (natural) laws and its description by mathematical ("transcendental") (PDE) models:

axiom option 1: mathematics is a construct of human mind, i.e. mathematics is an invention of mind (with its consequences to Schrödinger's arithmetic paradox (ScE)

axiom option 2: mathematics exists independently from human existence, i.e. mathematics is a discovery of mind.

At a first glance, the first option might sound somehow disappointingly, especially perhaps for mathematicians, but in the light of Gödel's result, it might turn around to be in fact good news, at a second glance! It is related to Kant's *Critique of Pure Reason* and model theory is a follow-up invention driven by rationality, not by mind (according to Kant), of course, with the risk of antinomies.

For axiom option 2 in his *Critique of Judgement* Kant puts mathematics as an own category to deduce from the general to the specific with respect to two aspects: an esthetical and teleological perspective. In other words, mathematics is a characteristic of an esthetical and fitness for purpose principle of Nature.