The proposed RH solution is in sync with the (quick) distributional way to prove the Prime Number Theorem (PNT) (J. Vindas, R. Estrada)

The prime number theorem describes the asymptotic distribution of the prime numbers.

Abel's method is concerned with the summation of divergent series, based on the Abelian theorem, that if the infinite series SUM(a(n)) is convergent with limit A, then the power series SUM((a(n)r*exp(n)) is convergent with same limit A, as t --> 1. The converse is, of course, not true, which forms the basis of the Abelian/Tauberian theory.

In J. Vindas, R. Estrada, "*A quick distributional way to the prime number theorem*", the theorem is proven using distribution theory, **whereby no Tauberian results are employed**:

http://cage.ugent.be/~jvindas/Publications_files/PNT.pdf

The standard approach (like Ikehara's proof) applies Tauberian theorems, whereby the prime number theorem is the approximation of the von Mangoldt's Stieltjes integral density function (d(psi(x)) ~ dx in a certain sense.

From H. M. Edwards, "Riemann's Zeta Function", chapter 12.7,

Edwards H. M., Riemann s Zeta Function, section 12.7, Tauberian Theorems

we recall:

"*perhaps the simplest formulation of the idea of the prime number theorem is the approximation of the von Mangoldt's Stieltjes integral density function (d(psi(x)) ~ dx. The Tauberian theorems give a natural interpretation of the approximate formula d(psi(x))~dx and shows a direct heuristic connection between it and the simple pole of Zeta(s) at s=1*." ...In this context the Ikehara's theorem (12.7) fits into. "*The original proof of this theorem was a deduction from Wiener's general Tauberian theorem*."

Ikehara's proof requires additional convergence property for Re(s)>1 and s --> +1 with respect to its application to the prime number theorem. "E*ssentially the same techniques as those used by Hadamard and de la Valée Poussin need to be applied".*

The idea of an only weak (Mellin) integral transforms representation of the Riemann duality equation (with its underlying duality concerning the exchange of s <--> (1-s), the converge Mellin integrals within (!) the critical stripe and a valid limit s --> 1 from insight the critical stripe (Re(s)<1 (!))) in the framework of complex valued distribution theory might enable a direct application of Tauberian theorems w/o any additional assumptions, as in case of the proof of the Ikehara theorem.

For the generalized Fourier coefficients a(n) of an element of the Hilbert scale H(-1/2) it holds the weak "one side" Tauberian condition. Standard Hilbert scale analysis in combination with the analysis from the paper "A note of the Bagchi formulation of the Nyman RH criterion" leads to the following proposition:

the prime number theorem can be proven in a weak sense within the H(-1/2) distribution framework (with similar arguments as J. Vindas/ R. Estrada), whereby basically the "Abel summability" condition (Hardy-Littlewood condition O(1/n)) is evident for elements of H(-1/2). Then standard density arguments prove the prime number theorem in a strong sense.

We emphasis, that the distributional approach of J. Vindas, R. Estrada provides a Hilbert scale framework (of basically L(2) functions) in contrast to current L(1) Banach space environment, which builds the basis for Wiener's famous General Tauberian Theorem (see B. E. Petersen below). This theorem is basically about an "if and only if" density characterization requiring non-vanishing constant Fourier terms. In contrast to this, the Hilbert transform of a L(2) function always has a vanishing constant Fourier terms, i.e. the weak distributional approach "bypasses" required additional "Tauberian" (convergence)conditions (which is the primarily purpose/intention of Tauberian theorems) to ensure convergent series and integrals.

The above leads also back to a still answered question of B. Riemann concerning triginometric series ((LaD) 2.2) about the representation of a given function f(x) by a trigonometric series and (more interesting) vice versa (where the discussed distributional Hilbert space framework now provides a solution option!):

"*Wenn eine Funktion durch eine trigonometrische Reihe darstellbar ist, was folgt daraus über ihren Gang, über die Änderung ihres Werthes bei stetiger Änderung der Arguments*?"

p. 193: "Ein denkbares Ziel, das bis heute nicht befriedigend erreicht ist, wäre: Man finde notwendige und hinreichende Bedingungen für reelle 2*pi-periodische f(x), so dass diese für alle reellen x gleich ihrer FOURIERschen Reihe oder überhaupt gleich einer trigonometrischen Reihe ist."

(LaD) Laugwitz D., "*Bernhard Riemann 1826-1966*", Birkhäuser Verlag, Basel, Boston, Berlin, 1996

At the same time, this approach enables the building of appropriate convolution operators (in a weak sense) according to the question 7 in the paper of D. Cardon below:

http://people.oregonstate.edu/~peterseb/misc/docs/abelian_and_tauberian_theorems.pdf

http://fuchsbraun.homepage.t-online.de/media/a3afe4d9e62f9d68ffff810effffffef.pdf .

Some further references about Tauberian theorems:

a brief introduction to some of the more important Tauberian theorems (including mercerian theorems as limiting cases) and the methods which have been developed to prove them are the aim of the book

H. R. Pitt, Tauberian Theorems, Oxford University Press, 1958:

Mercerian theorems are a generalization of Tauber theorems (which are buil on Abelian integral transformations), building on "**Stieljes kernel**" (H. R. Pitt,1.3). Theorem 3 (H.R. Pitt, 5.1) states a uniqueness theorem for a uniquely defined normalized bounded variation (density) function k(x), determined by a given function K(t) expressed in the Stieltjes form

K(t)=Int(exp(-itx)dk(x).

The correspondence between the conceptual idea of our two proof of the Riemann hypothesis and the Tauberian theorem related to the RH itself (as its stated e.g. in H. M.Edwards, "Riemann's Zeta Function, 12.7) is given by the results of:

Pilipovic S., Stankovic, Tauberian theorems for integral transforms of distributions, Acta Mat. Hungar. 74 (1-2) 135-153, 1997:

Pilipovic S., Stankovic B., Tauberian Theorems for Integral Transforms of Distributions

Borwein D., A Century of Tauberian Theory

Petersen B. E., Abelian and Tauberian Theorems, Philosophy

Trudgian T. An introduction to Tauberian theorems

Garding L., The Mathematics of Wiener's Tauberian theorem

Bingham N. H., Inoue, An Abel-Tauber Theorem for the Hankel Transforms

Paneva-Konovska J., Theorems on the convergence of series in generalized Lommel-Wright functions

Pilipovic S. Stankovic B., Tauberian Theorems for Integral Transforms of Distributions