SPACE—TIME—MATTER

INTRODUCTION

SPACE and **time** are commonly regarded as the **forms** of existence of the real world, **matter** as its **substance.** A definite portion of matter occupies a definite part of space at a definite moment of time. It is in the composite idea of **motion** that these three fundamental conceptions enter into intimate relationship. Descartes defined the objective of the exact sciences as consisting in the description of all happening in terms of these three fundamental conceptions, thus referring them to motion. Since the human mind first wakened from slumber, and was allowed to give itself free rein, it has never ceased to feel the profoundly mysterious nature of time-consciousness, of the progression of the world in time,—of Becoming. It is one of those ultimate metaphysical problems which philosophy has striven to elucidate and unravel at every stage of its history. The Greeks made Space the subject-matter of a science of supreme simplicity and certainty. Out of it grew, in the mind of classical antiquity, the idea of pure science. Geometry became one of the most powerful expressions of that sovereignty of the intellect that inspired the thought of those times. At a later epoch, when the intellectual despotism of the Church, which had been maintained through the Middle Ages, had crumbled, and a wave of scepticism threatened to sweep away all that had seemed most fixed, those who believed in Truth clung to Geometry as to a rock, and it was the highest ideal of every scientist to carry on his science ***" more geometrico "***. Matter was imagined to be a substance involved in every change, and it was thought that every piece of matter could be measured as a quantity, and that its characteristic expression as a "substance" was the Law of Conservation of Matter which asserts that matter remains constant in amount throughout every change. This, which has hitherto represented our knowledge of space and matter, and which was in many quarters claimed by philosophers as *a priori* knowledge, absolutely general and necessary, stands to-day a tottering structure. First, the physicists in the persons of Faraday and Maxwell, proposed the " electromagnetic **field** " in contradistinction to **matter,** as a reality of a different category. Then, during the last century, the mathematician, following a different line of thought, secretly undermined belief in the evidence of Euclidean Geometry. And now, in our time, there has been unloosed a cataclysm which has swept away space, time, and matter hitherto regarded as the firmest pillars of natural science, but only to make place for a view of things of wider scope, and entailing a deeper vision.

This revolution was promoted essentially by the thought of one man, Albert Einstein. The working-out of the fundamental ideas seems, at the present time, to have reached a certain conclusion; yet, whether or not we are already faced with a new state of affairs, we feel ourselvès compelled to subject these new ideas to a close analysis. Nor is any retreat possible. The development of scientific thought may once again take us beyond the present achievement, but a return to the old narrow and restricted scheme is out of the question.

Philosophy, mathematics, and physics have each a share in the problems presented here. We shall, however, be concerned above all with the mathematical and physical aspect of these questions. I shall only touch lightly on the philosophical implications for the simple reason that in this direction nothing final has yet been reached, and that for my own part I am not in a position to give such answers to the epistemological questions involved as my conscience would allow me to uphold. The ideas to be worked out in this book are not the result of some speculative inquiry into the foundations of physical knowledge, but have been developed in the ordinary course of the handling of concrete physical problems—problems arising in the rapid development of science which has, as it were, burst its old shell, now become too narrow. This revision of fundamental principles was only undertaken later, and then only to the extent necessitated by the newly formulated ideas. As things are to-day, there is left no alternative but that the separate sciences should each proceed along these lines dogmatically, that is to say, should follow in good faith the paths along which they are led by reasonable motives proper to their own peculiar methods and special limitations. The task of shedding philosophic light on to these questions is none the less an important one, because it is radically different from that which falls to the lot of individual sciences. This is the point at which the

philosopher must exercise his discretion. If he keep in view the boundary lines determined by the difficulties inherent in these problems, he may direct, but must not impede, the advance of sciences whose field of inquiry is confined to the domain of concrete objects.

Nevertheless I shall begin with a few reflections of a philosophical character. As human beings engaged in the ordinary activities of our daily lives, we find ourselves confronted in our acts of perception by material things. We ascribe a "real" existence to them, and we accept them in general as constituted, shaped, and coloured in such and such a way, and so forth, as they appear to us in our perception in "general," that is ruling out possible illusions, mirages, dreams, and hallucinations.

These material things are immersed in, and transfused by, a manifold, indefinite in outline, of analogous realities which unite to form a single ever-present world of space to which I, with my own body, belong. Let us here consider only these bodily objects, and not all the other things of a different category, with which we as ordinary beings are confronted; living creatures, persons, objects of daily use, values, such entities as state, right, language, etc. Philosophical reflection probably begins in every one of us who is endowed with an abstract turn of mind when he first becomes sceptical about the world-view of naïve realism to which I have briefly alluded.

It is easily seen that such a quality as "**green**" has an existence only as the correlate of the sensation "green" associated with an object given by perception, but that it is meaningless to attach it as a thing in itself to material things existing **in themselves.** This recognition of the **subjectivity of the qualities of sense** is found in Galilei (and also in Descartes and Hobbes) in a form closely related to thè principle underlying the **constructive mathematical method of our modern physics which repudiates "qualities".** According to this principle, colours are "really" vibrations of the æther, i.e. motions. In the field of philosophy Kant was the first to take the next decisive step towards the point of view that not only the qualities revealed by the senses, but also space and spatial characteristics have no objective significance in the absolute sense; in other words, that **space, too, is only a form of our perception.** In the realm of physics it is perhaps only the theory of relativity which has made it quite clear that the two essences, space and time, entering into our intuition have no place in the world constructed by mathematical physics. Colours are thus "really" not even æther-vibrations, but merely a series of values of mathematical functions in which occur four independent parameters corresponding to the three dimensions of space, and the one of time.

Expressed as a general principle, this means that the real world, and every one of its constituents with their accompanying characteristics, are, and can only be given as, intentional objects of acts of consciousness. The immediate data which I receive are the experiences of consciousness in just the form in which I receive them. They are not composed of the mere stuff of perception, as many Positivists assert, but we may say that in a sensation an object, for example, is actually physically present for me—to whom that sensation relates—in a manner known to every one, yet, since it is characteristic, it cannot be described more fully. Following Brentano, I shall call it the "**intentional object**". In experiencing perceptions I see this chair, for example. My attention is fully directed towards it. I "have" the perception, but it is only when I make this perception in turn the intentional object of a new inner perception (a free act of reflection enables me to do this) that I "know" something regarding it (and not the chair alone), and ascertain precisely what I remarked just above. In this second act the intentional object is immanent, i.e. like the act itself, it is a real component of my stream of experiences, whereas in the primary act of perception the object is transcendental, i.e. it is given in an experience of consciousness, but is not a real component of it. What is immanent is **absolute,** i.e. it is exactly what it is in the form in which I have it, and I can reduce this, its essence, to the axiomatic by acts of reflection. On the other hand, transcendental objects have only a **phenomenal** existence; they are appearances presenting themselves in manifold ways and in manifold "gradations". One and the same leaf seems to have such and such a size, or to be coloured in such and such a way, according to my position and the conditions of illumination. Neither of these modes of appearance can claim to present the leaf just as it is "in itself". Furthermore, in every perception there is, without doubt, involved the **thesis of reality** of the object appearing in it; the latter is, indeed, a fixed and lasting element of the general thesis of reality of the world. When, however, we pass from the natural view to the philosophical attitude, meditating upon perception, we no longer subscribe to this thesis. We simply affirm that something real is "supposed" in it. The meaning of such a supposition now becomes the problem which must be solved from the data of consciousness. In addition a justifiable ground for making it must be found. I do not by this

in any way wish to imply that the view that the events of the world are a mere play of the consciousness produced by the ego, contains a higher degree of truth than naïve realism; on the contrary, we are only concerned in seeing clearly that the datum of consciousness is the starting-point at which we must place ourselves if we are to understand the absolute meaning as well as the right to the supposition of reality. In the field of logic we have an analogous case. A judgment, which I pronounce, affirms a certain set of circumstances; it takes them as true. Here, again, the philosophical question of the meaning of, and the justification for, this thesis of truth arises; here, again, the idea of objective truth is not denied, but becomes a problem which has to be grasped from what is given absolutely. "Pure consciousness" is the seat of that which is philosophically *a priori*. On the other hand, a philosophic examination of the thesis of truth must and will lead to the conclusion that none of these acts of perception, memory, etc., which present experiences from which I seize reality, gives us a conclusive right to ascribe to the perceived object an existence and a constitution as perceived. This right can always in its turn be over-ridden by rights founded on other perceptions, etc.

It is the nature of a real thing to be inexhaustible in content; we can get an ever deeper insight into this content by the continual addition of new experiences, partly in apparent contradiction, by bringing them into harmony with one another. In this interpretation, things of the real world are approximate ideas. From this arises the empirical character of all our knowledge of reality.*

Time is the primitive form of the stream of consciousness. It is a fact, however obscure and perplexing to our minds, that the contents of consciousness do not present themselves simply as being (such as conceptions, numbers, etc.), but as **being now** filling the form of the enduring present with a varying content. So that one does not say this **is** but this is **now**, yet now no more. If we project ourselves outside the stream of consciousness and represent its content as an object, it becomes an event happening in time, the separate stages of which stand to one another in the relations of **earlier** and **later.**

Just as time is the form of the stream of consciousness, so one may justifiably assert that space is the form of external material reality. All characteristics of material things as they are presented to us in the acts of external perception (e.g. colour) are endowed with the separateness of spatial extension, but it is only when we build up a single connected real world out of all our experiences that the spatial extension, which is a constituent of every

* Note 1.

perception, becomes a part of one and the same all-inclusive space. Thus space is the **form** of the external world. That is to say, every material thing can, without changing content, equally well occupy a position in Space different from its present one. This immediately gives us the property of the homogeneity of space which is the root of the conception, Congruence.

Now, if the worlds of consciousness and of transcendental reality were totally different from one another, or, rather, if only the passive act of perception bridged the gulf between them, the state of affairs would remain as I have just represented it, namely, on the one hand a consciousness rolling on in the form of a lasting present, yet spaceless; on the other, a reality spatially extended, yet timeless, of which the former contains but a varying appearance. Antecedent to all perception there is in us the experience of effort and of opposition, of being active and being passive. For a person leading a natural life of activity, perception serves above all to place clearly before his consciousness the definite point of attack of the action he wills, and the source of the opposition to it. As the doer and endurer of actions I become a single individual with a psychical reality attached to a body which has its place in space among the material things of the external world, and by which I am in communication with other similar individuals. Consciousness, without surrendering its immanence, becomes a piece of reality, becomes this particular person, namely myself, who was born and will die. Moreover, as a result of this, consciousness spreads out its web, in the form of time, over reality. Change, motion, elapse of time, becoming and ceasing to be, exist in time itself; just as my will acts on the external world through and beyond my body as a motive power, so the external world is in its turn **active** (as the German word "Wirklichkeit," reality, derived from "wirken" = to act, indicates). Its phenomena are related throughout by a **causal connection.** In fact physics shows that cosmic time and physical form cannot be dissociated from one another. The new solution of the problem of amalgamating space and time offered by the theory of relativity brings with it a deeper insight into the harmony of action in the world.

The course of our future line of argument is thus clearly outlined. What remains to be said of time, treated separately, and of grasping it mathematically and conceptually may be included in this introduction. We shall have to deal with space at much greater length. Chapter I will be devoted to a discussion of **Euclidean space** and its mathematical structure. In Chapter II will be developed those ideas which compel us to pass beyond the

Euclidean scheme; this reaches its climax in the general space-conception of the **metrical continuum** (Riemann's conception of space). Following upon this Chapter III will discuss the problem mentioned just above of the **amalgamation** of Space and Time in the world. From this point on the results of mechanics and physics will play an important part, inasmuch as this problem by its very nature, as has already been remarked, comes into our view of the world as an active entity. The edifice constructed out of the ideas contained in Chapters II and III will then in the final Chapter IV lead us to Einstein's *General Theory of Relativity*, which, physically, entails a new Theory of **Gravitation,** and also to an extension of the latter which embraces electromagnetic phenomena in addition to gravitation. The revolutions which are brought about in our notions of Space and Time will of necessity affect the conception of matter too. Accordingly, all that has to be said about matter will be dealt with appropriately in Chapters III and IV.

To be able to apply mathematical conceptions to questions of Time we must postulate that it is theoretically possible to fix in Time, to any order of accuracy, an absolutely rigorous **now** (present) as a **point of Time**—i.e. to be able to indicate points of time, one of which will always be the earlier and the other the later. The following principle will hold for this "order-relation". If A is earlier than B and B is earlier than C, then A is earlier than C. Each two points of Time, A and B, of which A is the earlier, mark off a **length of time;** this includes every point which is later than A and earlier than B. The fact that Time is a form of our stream of experience is expressed in the idea of **equality:** the empirical content which fills the length of Time AB can in itself be put into any other time without being in any way different from what it is. The length of time which it would then occupy is equal to the distance AB. This, with the help of the principle of causality, gives us the following objective criterion in physics for equal lengths of time. If an absolutely isolated physical system (i.e. one not subject to external influences) reverts once again to exactly the same state as that in which it was at some earlier instant, then the same succession of states will be repeated in time and the whole series of events will constitute a cycle. In general such a system is called a **clock.** Each period of the cycle lasts **equally** long.

The mathematical fixing of time by **measuring** it is based upon these two relations, "earlier (or later) times" and "equal times". The nature of measurement may be indicated briefly as follows:

Time is homogeneous, i.e. a single point of time can only be given by being specified individually. There is no inherent property arising from the general nature of time which may be ascribed to any one point but not to any other; or, every property logically derivable from these two fundamental relations belongs either to all points or to none. The same holds for time-lengths and point-pairs. A property which is based on these two relations and which holds for **one** point-pair must hold for every point-pair AB (in which A is earlier than B). A difference arises, however, in the case of three point-pairs. If any two time-points O and E are given such that O is earlier than E, it is possible to fix conceptually further time-points P by referring them to the unit-distance OE. This is done by constructing logically a relation t between three points such that for every two points O and E, of which O is the earlier, there is one and only one point P which satisfies the relation t between O, E and P, i.e. symbolically,

$$OP = t \, . \, OE$$

(e.g. $OP = 2 \, . \, OE$ denotes the relation $OE = EP$). **Numbers** are merely concise symbols for such relations as t, defined logically from the primary relations. P is the "time-point with the **abscissa t in the co-ordinate system** (taking OE as unit length)". Two different numbers t and t^* in the same co-ordinate system necessarily lead to two different points; for, otherwise, in consequence of the homogeneity of the continuum of time-lengths, the property expressed by

$$t \, . \, AB = t^* \, . \, AB,$$

since it belongs to the time-length $AB = OE$, must belong to **every** time-length, and hence the equations $AC = t \, . \, AB$, $AC = t^* \, . \, AB$ would both express the same relation, i.e. t would be equal to t^*. Numbers enable us to single out separate time-points relatively to a unit-distance OE out of the time-continuum by a conceptual, and hence objective and precise, process. But the objectivity of things conferred by the exclusion of the ego and its data derived directly from intuition, is not entirely satisfactory; the co-ordinate system which can only be specified by an individual act (and then only approximately) remains as an inevitable residuum of this elimination of the percipient.

It seems to me that by formulating the principle of measurement in the above terms we see clearly how mathematics has come to play its rôle in exact natural science. *An essential feature of measurement is the difference between the "determination" of an object by individual specification and the determination of the same object by some conceptual means.* The latter is only possible relatively to objects which must be defined directly. That is why a **theory of relativity** is perforce always involved in measurement. The general problem which it proposes for an arbitrary domain of objects takes the form: (1) What must be given such that relatively to it (and to any desired order of precision) one can single out conceptually a single arbitrary object P from the continuously extended domain of objects under consideration? That which has to be given is called the **co-ordinate system**, the conceptual definition is called the **co-ordinate** (or abscissa) of P in the co-ordinate system. Two different co-ordinate systems are completely equivalent for an objective standpoint. There is no property, that can be fixed conceptually, which applies to one co-ordinate system but not to the other; for in that case too much would have been given directly. (2) What relationship exists between the co-ordinates of one and the same arbitrary object P in two different co-ordinate systems?

In the realm of time-points, with which we are at present concerned, the answer to the first question is that the co-ordinate system consists of a time-length OE (giving the origin and the unit of measure). The answer to the second question is that the required relationship is expressed by the formula of transformation

$$t = at' + b \qquad (a > o)$$

in which a and b are constants, whilst t and t' are the co-ordinates of the same arbitrary point P in an "unaccented" and "accented" system respectively. For all possible pairs of co-ordinate systems the characteristic numbers, a and b, of the transformation may be any real numbers with the limitation that a must always be positive. The aggregate of transformations constitutes a **group,** as their nature would imply, i.e.,

1. "identity" $t = t'$ is contained in it.

2. Every transformation is accompanied by its reciprocal in the group, i.e. by the transformation which exactly cancels its effect. Thus, the inverse of the transformation (a, b), viz. $t = at' + b$, is $\left(\frac{1}{a}, -\frac{b}{a}\right)$, viz. $t' = \frac{1}{a}t' - \frac{b}{a}$.

3. If two transformations of a group are given, then the one which is produced by applying these two successively also belongs to the group. It is at once evident that, by applying the two transformations

$$t = at' + b \qquad t' = a't'' + b'$$

in succession, we get

$$t = a_1 t'' + b_1$$

where $a_1 = a \,.\, a'$ and $b_1 = (ab') + b$; and if a and a' are positive, so is their product.

The theory of relativity discussed in Chapters III and IV proposes the problem of relativity, not only for time-points, but for the physical world in its entirety. We find, however, that this problem is solved once a solution has been found for it in the case of the two forms of this world, space and time. By choosing a co-ordinate system for space and time, we may also fix the physically real content of the world conceptually in all its parts by means of numbers.

All beginnings are obscure. Inasmuch as the mathematician operates with his conceptions along strict and formal lines, he, above all, must be reminded from time to time that the origins of things lie in greater depths than those to which his methods enable him to descend. Beyond the knowledge gained from the individual sciences, there remains the task of **comprehending.** In spite of the fact that the views of philosophy sway from one system to another, we cannot dispense with it unless we are to convert knowledge into a meaningless chaos.

CHAPTER I

EUCLIDEAN SPACE. ITS MATHEMATICAL FORMULATION AND ITS RÔLE IN PHYSICS

§ 1. Deduction of the Elementary Conceptions of Space from that of Equality

JUST as we fixed the present moment (" now ") as a geometrical point in time, so we fix an exact " here," a point in space, as the first element of continuous spatial extension, which, like time, is infinitely divisible. Space is not a one-dimensional continuum like time. The principle by which it is continuously extended cannot be reduced to the simple relation of " earlier " or " later ". We shall refrain from inquiring what relations enable us to grasp this continuity conceptually. On the other hand, space, like time, is a **form** of phenomena. Precisely the same content, identically the same thing, still remaining what it is, can equally well be at some place in space other than that at which it is actually. The new portion of Space **S**′ then occupied by it is equal to that portion **S** which it actually occupied. **S** and **S**′ are said to be **congruent.** To every point P of **S** there corresponds one definite **homologous** point P' of **S**′ which, after the above displacement to a new position, would be surrounded by exactly the same part of the given content as that which surrounded P originally. We shall call this " transformation " (in virtue of which the point P' corresponds to the point P) a **congruent transformation.** Provided that the appropriate subjective conditions are satisfied the given material thing would seem to us after the displacement exactly the same as before. There is reasonable justification for believing that a rigid body, when placed in two positions successively, realises this idea of the equality of two portions of space; by a **rigid** body we mean one which, however it be moved or treated, can always be made to appear the same to us as before, if we take up the appropriate position with respect to it. I shall evolve the scheme of geometry from the conception of equality combined with that of continuous connection—of which the latter offers great difficulties to analysis