

Introduction

1.1 Turbulence and symmetries

In Chapter 41 of his *Lectures on Physics*, devoted to hydrodynamics and turbulence, Richard Feynman (1964) observes this:

Often, people in some unjustified fear of physics say you can't write an equation for life. Well, perhaps we can. As a matter of fact, we very possibly already have the equation to a sufficient approximation when we write the equation of quantum mechanics:

$$H\psi = -\frac{\hbar}{i} \frac{\partial \psi}{\partial t}. \quad (1.1)$$

Of course, if we only had this equation, without detailed observation of biological phenomena, we would be unable to reconstruct them. Feynman believes, and this author shares his viewpoint, that an analogous situation prevails in *turbulent* flow of an incompressible fluid. The equation, generally referred to as the Navier–Stokes equation, has been known since Navier (1823):

$$\partial_t \mathbf{v} + \mathbf{v} \cdot \nabla \mathbf{v} = -\nabla p + \nu \nabla^2 \mathbf{v}, \quad (1.2)$$

$$\nabla \cdot \mathbf{v} = 0. \quad (1.3)$$

It must be supplemented by initial and boundary conditions (such as the vanishing of \mathbf{v} at rigid walls). We shall come back later to the choice of notation.

The Navier–Stokes equation probably contains all of turbulence. Yet it would be foolish to try to guess what its consequences are without looking at experimental facts. The phenomena are almost as varied as in the realm of life.

TURBULENCE

THE LEGACY OF A. N. KOLMOGOROV

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