In a nutshell:
The solution concept to prove the RH

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The Riemann Hypothesis states that the non-trivial zeros of the Zeta function all have real part one-half. The Hilbert-Polya conjecture states that the imaginary parts of the zeros of the Zeta function corresponds to eigenvalues of an unbounded self-adjoint operator. All attempts failed so far to represent the Riemann duality equation in the critical stripe as convergent Mellin transforms of an underlying self-adjoint integral operator equation. The solution concept to answer the RH is about the following:

1. *building two self-adjoint, bounded (singular integral) operators of* $(s-1)\zeta(s)$ *on the critical line, which correspond to two convolution integral representation (*[CaD]) *with appropriate Hilbert space domains enabling proof P1 (Gaussian function based) and proof P2 (fractional part function based)*
2. *applying the special degenerated hypergeometric (Kummer) function ([GrI] 3.952, 7.612) with remarkable asymptotic of its zeros* [SeA] (*and, as option to model co-variant energy description of photons as* $(n+1/2)\hbar\omega$*)*

$$H[f](x)=2\int_0^\infty f(\xi)\sin(2\pi\xi x)d\xi=2xf(x)_1F_1(\frac{1}{2};\frac{3}{2},\pi x^2)=2x_1F_1(1;\frac{3}{2},-\pi x^2)$$

to define a modified integral exponential function (with slower decrease to infinity as the integral exponential function) and a modified Gamma functions (with more appreciated properties for $s\to 0$ *and* $s\to 1$*) by*

$$E(-x):=-\int_x^\infty e^{-t}d\log t \qquad \to \qquad E^*(-x):=2_1F_1'(\frac{1}{2};\frac{3}{2},-x)$$

$$\Gamma(\frac{s}{2})=\int_0^\infty x^{s/2}dE \qquad \to \qquad \Gamma^*(\frac{s}{2}):=\int_0^\infty x^{s/2}dE^*=\Gamma(\frac{s}{2})\frac{s}{s-1}=2\frac{\Pi(1+\frac{s}{2})}{s-1}.$$

Let H denotes the singular integral convolution Hilbert transform operator with domain $L_2(-\infty,\infty)$ and $f_H:=H[f]$ ([PeB]). Then for the Gaussian function f it holds in the neighborhood of $s=1$

$$M[f_H](s)=\frac{1}{2}\pi^{-s/2}\frac{\Gamma(\frac{1+s}{2})\Gamma(\frac{1-s}{2})}{\Gamma(1-s)} \quad \text{with} \quad f_H(x)=2xe^{-\pi x^2}{}_1F_1(\frac{1}{2};\frac{3}{2},\pi x^2).$$

The functions f_H and f are identical in a weak $L_2(-\infty,\infty)\cong l_2\cong l_2^0$ ensuring appropriate relationship to the RH theory ([EdH], [IvA], [TiE]). A similar approach with respect to the Hilbert space $L_2^\#(0,1)\cong l_2$ is valid for the fractional part function φ and its Hilbert transform φ_H:

$$\varphi(x):=2\pi(x-[x])=\pi-\sum_1^\infty\frac{2}{n}\sin 2\pi nx \,,\quad \varphi_H(x):=-2\log 2\sin(\pi x)=\sum_1^\infty\frac{2}{n}\cos 2\pi nx\,.$$

The special Kummer function enables the definition of an alternative Li-function with appreciated approximation properties in the form

$$Li^*(x):=2_1F_1(\frac{1}{2};\frac{3}{2},\log x)=\sum_0^\infty\frac{1}{n+1/2}\frac{\log^n x}{n!}=\int_0^1 x^t t^{-1/2}dt\,.$$

In [BaB] a l_2^α – Hilbert space reformulation of the Nyman-Beurling criterion is given ($\alpha=-1$).The Hilbert scale factor $\alpha=1/2$ is related to the Dirichlet integral of harmonic function over the unit circle e.g. in the context of conformal mapping and minimal surfaces theory ([CoR] (1.4)). The Hilbert scale factor $\alpha=-1/2$ is related to wavelet theory ([LoA]) on the unit disk (by Daubechies and Möbius wavelets). The reproducing property (Calderón's reproducing formula for the continuous wavelet transform) of Möbius wavelets is valid in a weak $H_{-1/2}$ – sense. The zeros σ_n of the Kummer function ${}_1F_1(2\pi ix)$ lie in the intervals $(n-1/2,n)$ [SeA]. Those properties are proposed to leverage the Hardy-Littlewood circle method to prove the binary Goldbach conjecture.

References

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