RH 2016 – 2017, page text

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The big challenge which jeopardizes a proof of the RH is about the not-vanishing constant Fourier term of the Gaussian function. The proposed new approach replacing the Gaussian function by its Hilbert transform, which is identical to the Dawson function F(x), solves this issue. Therefore, the solution concept is about a replacement of the even Gauss-Weierstrass function (GWF) and the even fractional part function (FPF) by its related odd Hilbert transforms:

H(GWF) = Dawson function = sin-integral transform of GWF

H(FPF) = log(2sinx)-function .

We note the following properties of the Hilbert transform:

1. H*H = - I

2. Hu, u are L(2)-norm equivalent and orthogonal, i.e. (u, Hu)=0

3. (xH-Hx)(v)(x) = 0 for all odd L(2)-functions v.

As a consequence the constant Fourier terms of both transforms now vanish, while the corresponding Theta function property (theta(1/x)=x*theta(x)) which is equivalent to the Riemann duality equation keeps preserved in a weak L(2)-sense. The Mellin transform of the Dawson function reflects this kind of symmetry along the critical line in contrast to the Mellin transform of the original Gauss-Weierstrass function which is basically the Gamma function. The concept replaces today's Banach space framework by a Hilbert space framework, while at the same time the GWF/FPF are replaced by their corresponding Hilbert transform, defining corresponding alternative (more appropriate) Mellin transforms.

The li(x)-function is based on the Ei(x)-function which is built on the Gauss-Weierstrass integral function. The Dawson integral function is proposed as an alternative Ei(x)-function, enabling an alternative li(x)-function with identical convergence behavior for x to infinity. The later one leads to a modified Riemann error function (Edwards H. M., 1.14).

With respect to the NSE and the YME topics we note that the Gauss-Weierstrass function is a model for Maxwell's velocity distributions of gases and the Brownian motion. Therefore, its Hilbert transform provides also an alternative model in the context of Maxwell and Yang-Mills equations. The generalization of the Hilbert transform for space dimensions n>1 leads to the Riesz transforms which are rotation invariant.

The Hilbert transformed GWF (the Dawson function, which is a special Kummer function) enables the definition of a singular self-adjoint (convolution) integral operator on the critical line, which is bounded in appropriately defined (distributional) Hilbert space domain (alternatively to Riemann's representation, Edwards H. M., §1.8).
Applying spectral theory this then proves the Hilbert-Polya conjecture and therefore, the RH.

A similar approach as for the Gaussian function is valid for the fractional part function and its related Hilbert transform (Titchmarsh E. C.).

The new theory enables also the validation of other RH criteria (e.g. the Nyman criterion (Bagchi formulation), the pi(x) - li(x) = O(square(x) * logx) criterion with alternative li-function definition, Polya criteria (Weierstrass product for entire functions of genus 0 or 1 (applied for Kummer functions) or Jensen criterion), Li's criteron with alternative entire Zeta function definition).

The Dawson function (=H(GWF))

We recall the essential properties of the Dawson function F(x):

i) F(0)=0 and its Fourier transform has a vanishing constant Fourier term

ii) Maximum point: 0.54<F(x(max))<0.55, whereby 0.92<x(max)<0.93

iii) Reflection point: 0.42<F(x(refl))<0.43, whereby 1.50<x(refl)<1.51

iv) F(x)=O(1/x) for x to infinity, i.e. a "linear"-polynomial decrease, only!

v) F(x) is directly related to the erf(x)-function, the Fresnel functions and a specific confluent hypergeometric functions

vi) The Dawson function is a Siegel E-function (as it is a hypergeometric function). A continued fraction expansion for the Dawson integral is given in

The Dawson function and its usage in an appropriate Hilbert scale (in combination with singular integral (Pseudo-Differential) Operators) is also proposed as enhanced modelling tool for open questions in quantum physics.

Its usage in other (mathematical or physical model) areas, where the Gaussian function is currently applied to with minor success, might provide opportunities, as well.

In general this relates to the different application areas of ("normal" and others) probability distribution functions. The essential corresponding Hilbert transform property to generate alternative distribution (density) functions is the following:

let f be a L(2) function and g:=H(f) denote its Hilbert transform. Then g is also a L(2) function, which is orthogonal to f with respect to the inner product of L(2), i.e. (f,g)=0.

P. Biane et.al. Probability laws related to Jacobi-Theta, Riemann Zeta functions …

Specifically this relates to the

- *heat equation* and evolution of a mass density under diffusion
- *Gaussian filter function* and signal processing- time-symmetric wave function and entropy
- *wave function*, Bose-Einstein statistics, ground state energy of quantum harmonic oscillator and vacuum state in quantum field theory
- ***Maxwell's velocity distributions*** of gases, matching quite well the Dawson function
- ***Brownian motion***, white noise and Mandelbrot fractals and its relation to

--> THE MUSIC OF THE PRIMES & SUBATOMIC PARTICLES

given an answer to Derbyshine's ("Prime Obsession") question...

... "*The non-trivial zeros of Riemann's zeta function arise from inquiries into the distribution of prime numbers. The eigenvalues of a random hermitian matrix arise from inquiries into the behavior of systems of subatomic particles under the laws of quantum mechanics. What on earth does the distribution of prime numbers have to do with the behavior of subatomic particles?*"

... and, what on earth does have the distribution of the prime numbers have to do with harmonic music?

The music of the primes, harmonic music noise between red and white noise and subatomic particles

The H(1/2) (energy) Hilbert space on the unit circle

The proposed alternative FPF Hilbert space framework and the harmonic music "noise" is related to the H(1/2) Hilbert space of periodic function on the circle. This Hilbert space is also successfully applied in other areas:

Biswas I., Nag S., Jacobians of Riemann Surfaces and the Sobolev H(1-2) on the Circle

Nag S., Sullivan D., Teichmueller theory and the universal period mapping via quantum calculus and the H(1 2) sapce on the circle

The H(1/2) Hilbert space plays also a key role concerning questions related to the topological degree of continuous maps resp. to the continuous cycle with its well defined winding number:

Bourgain J., Kozma G., One cannot hear the winding number

Brezis H., New questions related to the topological degree

Some opportunities might be given for "probability methods" in number theory, e.g. with respect to the distributions of additive-theoretical functions

Kac M., Probability methods in some problems of analysis and number theory